




Thurnby Mead Calculation Policy

Progression of Skills	How to do it
Addition	<p>Pre- requisite skills</p> <ul style="list-style-type: none">• Conservation of number (knowing a quantity doesn't change unless you add or takeaway)• 1:1 correspondence• Cardinality (knowing the last number states how many there are)• Subitising (no need to count)• Developing language – using and understanding the language before words are used)• Oral counting: Develop an unbreakable list<ul style="list-style-type: none">Break the chain – start counting from any numberCount on from the first numberCount on from the largest number
	<p>Recognise that there are more objects when one or more objects have been added</p> <p>Can use apparatus and use the language of more than e.g. use the Numicon to say that one more than 2 is 3</p>  <p>Can count on to find one more using fingers and counters</p>

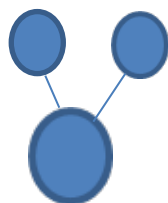


**Begin to relate addition
to combining two
groups of objects**

Use number tracks to find one more than

Use the circles model to act out addition.

Count out a group of objects in each smaller circle. Drag each group into the larger circle to find out how many there are altogether.



Children should have access to a wide range of counting equipment, everyday objects, number tracks, number lines and be shown in different contexts

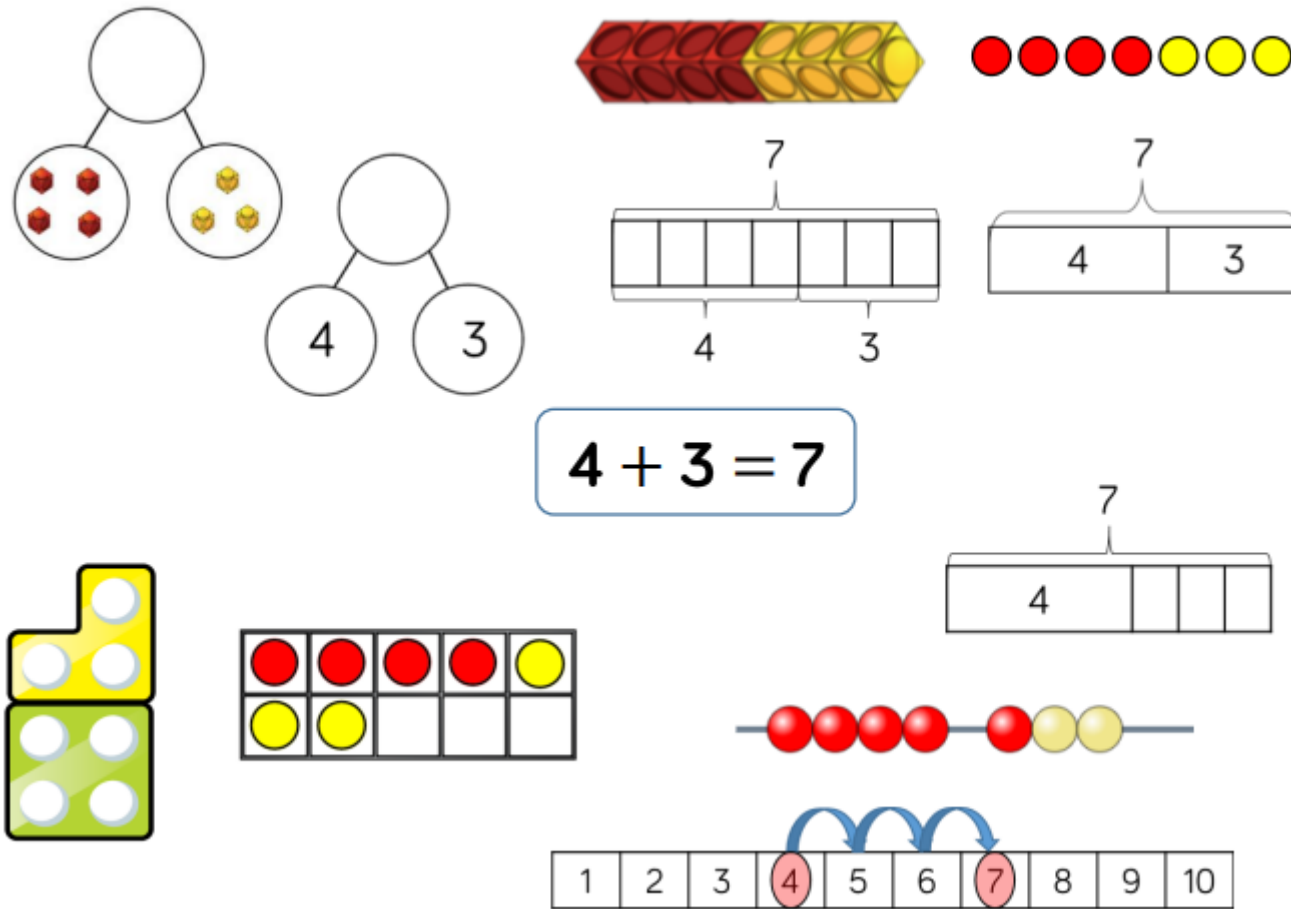
**Adding two 1 digit
numbers within 10**

$$4 + 3$$

Show this as a number story

Use equipment for both numbers and count all equipment





Count on from the first number

When adding numbers to 10, children can explore both aggregation and augmentation. The part-whole model, the discrete and continuous bar model, Numicon and ten frame support aggregation. The combination bar model, ten frame, bead string and number track all support augmentation.



Count on from the largest number

Use equipment for both numbers and count on from the first number

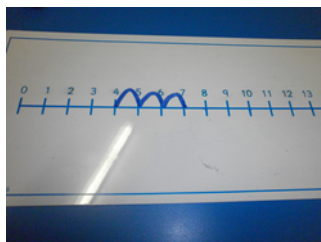
Use equipment for both numbers and count on from the biggest number

Highlight the biggest number and get the equipment for the other number and count on.

Use a number track



Use a number line



Demonstrate to the children how the objects they have been counting with now relate to the number line. A Numicon number line would be good for this.

Rapid recall of number bonds

The children should read and write the addition and equals signs within the number sentences. The children need to understand that equals means balances – whatever is either side of the equals sign is the same – it balances. The children need experience of seeing the = sign at different positions in the calculations.



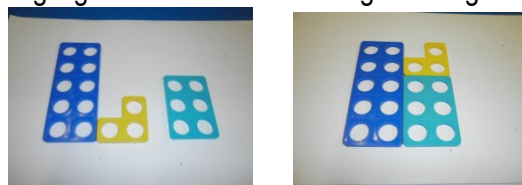
Adding a 2 digit and a 1 digit number

Quick mental recall of addition number facts is essential: Number bonds within 10 and number bonds to 10, doubles to 5+5 and 10+10

Not bridging tens

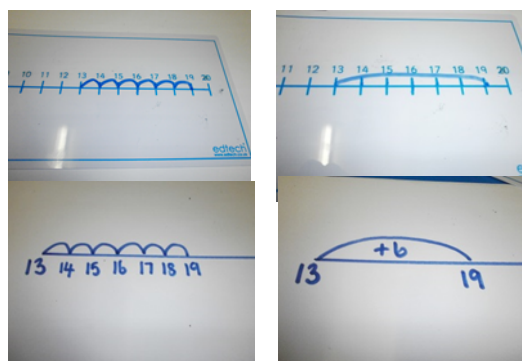
$$6 + 13$$

Use equipment starting on the largest number
Highlight the link for the unit digit making known bonds



Highlight the starting number – only use equipment for the smaller number and count on

Use a printed number line – jump in steps of 1 or a jump of 6



Use a blank number line – jump in steps of 1 or a jump of 6

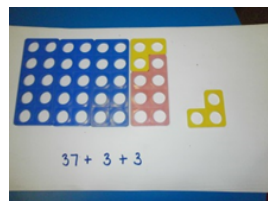
Numbers that bridge 10



Adding a 2 digit and a 1 digit number

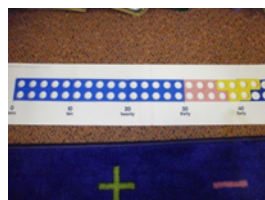
$$6 + 37$$

Use the equipment e.g. Numicon or base 10

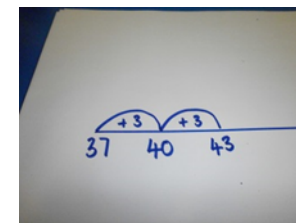


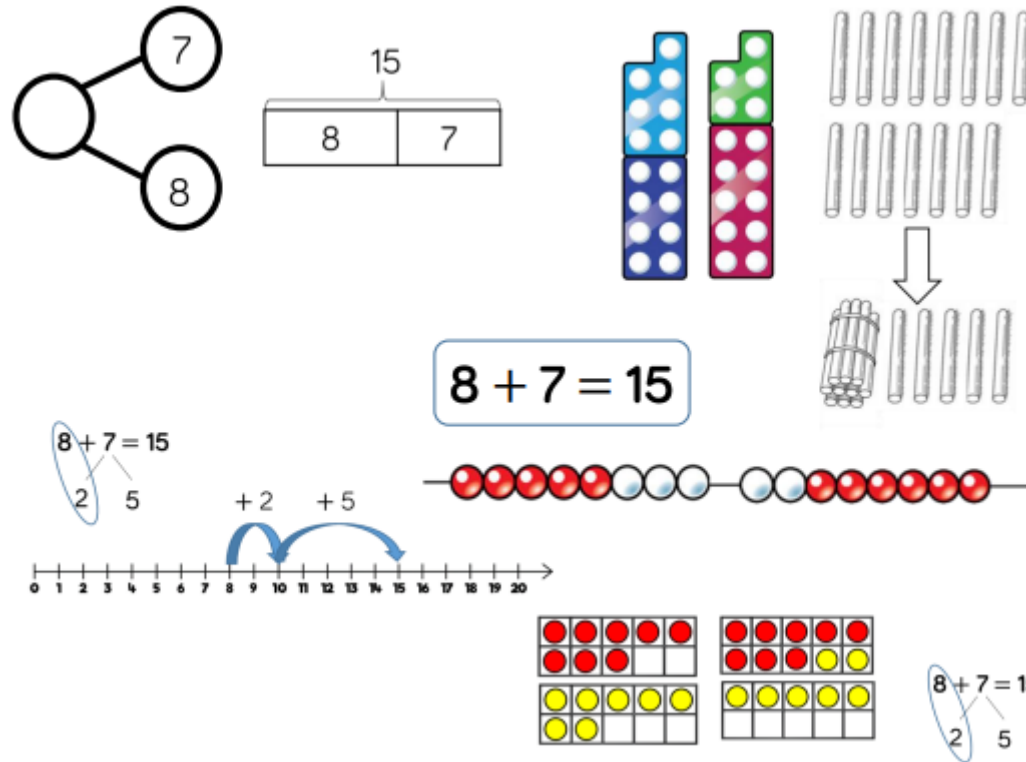
Use the equipment for the last number only – how would we snap the number to get to the next multiple of 10?

Efficient jumps on a printed number line



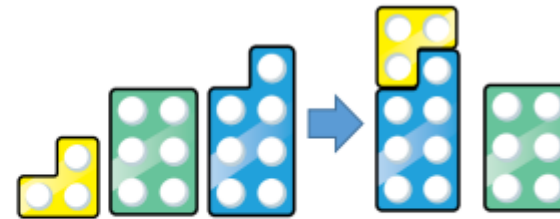
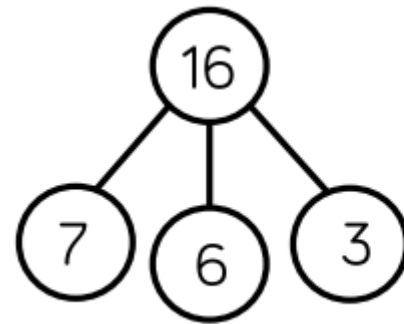
Drawing your own number line and doing it in 2 jumps – using known bonds to support this.



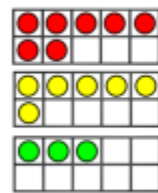


Adding 3 1-digit numbers

When adding 1-digit numbers that cross 10, it is important to highlight the importance of 10 ones equalling one ten. Different manipulatives can be used to represent this exchange. Use concrete resources alongside number lines to support children in understanding how to partition their jumps.

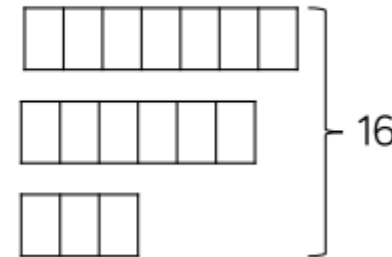


$$7 + 6 + 3 = 16$$



$$7 + 6 + 3 = 16$$

10



Adding two 2-digit numbers

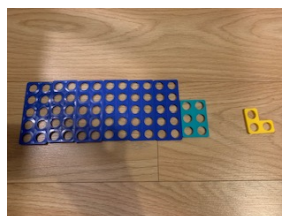
When adding 3 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently. This supports the children in their understanding of commutativity. Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

Numbers that don't bridge 10



$$23 + 46$$

Using structured place value apparatus for both numbers



Only use equipment for the smallest number and count on with the tens first and then the ones

Use a 100 square – adding the tens first and then the ones

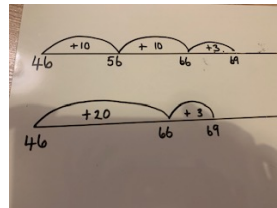
0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Drawing your own number line, adding the tens in 2 jumps of 10 then add the ones.



Adding two 2 digit numbers

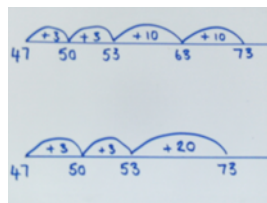
Drawing your own number line, adding a jump of 20 and then add the ones



Numbers that bridge 10

$$26 + 47$$

Use equipment for both numbers



Drawing your own number line and show the jumps – tens first and then the ones



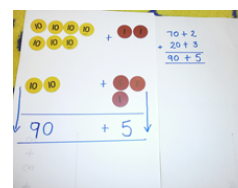
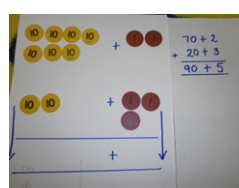
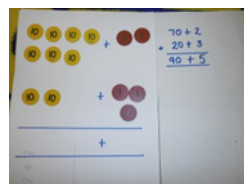
Adding pairs of 2 digit numbers using the partitioned column method

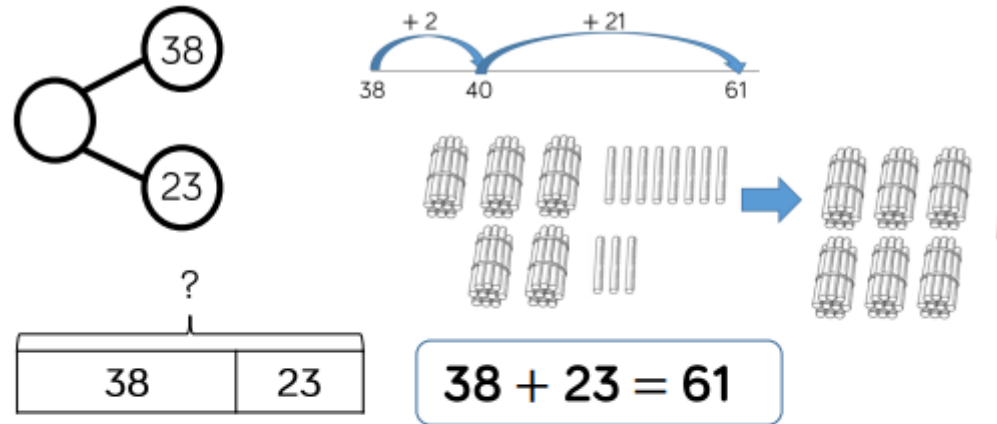
Numbers that don't and do bridge 10

$$72 + 23$$

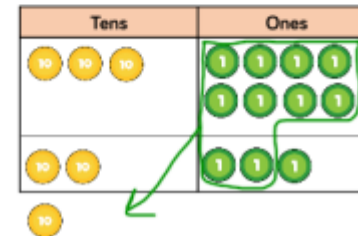
Partitioned column method

To support understanding, pupils should physically make and carry out the calculation with equipment, then compare their practical version to the written form, to help them build an understanding of it.





$$\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ 1 \end{array}$$



Adding numbers with up to 3 digits

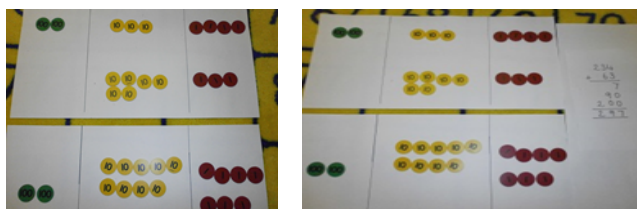
At this stage encourage the children to use the formal column method when calculated alongside straws, base-ten or place value counters. As numbers become larger, straws become less efficient.

Expanded column method

$$234 + 63$$



$$\begin{array}{r} 234 \\ + 63 \\ \hline 7 \\ 90 \\ 200 \\ \hline 297 \end{array}$$



Show this method alongside place value equipment so the children can see the link

$$247 + 125$$

$$\begin{array}{r} 247 \\ + 125 \\ \hline 12 \\ 60 \\ 300 \\ \hline \end{array}$$



372



Doing it with equipment will lead the children to understand the exchange between the tens and the ones

Children who are very secure and confident with 3 digit expanded column addition should be moved onto the compact column addition method, being introduced to carrying for the first time. Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

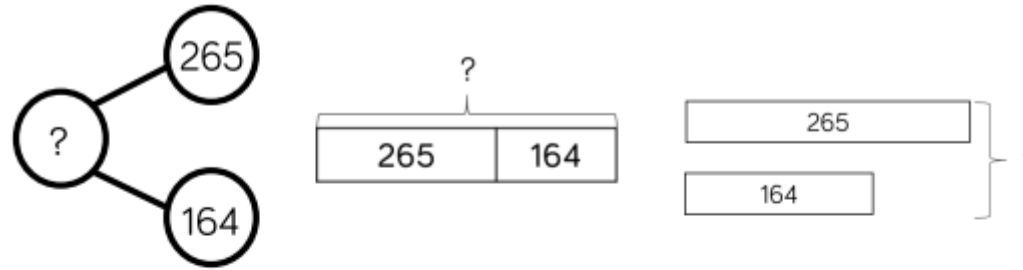
$$246 + 138$$

$$\begin{array}{r} 246 \\ + 138 \\ \hline \end{array}$$

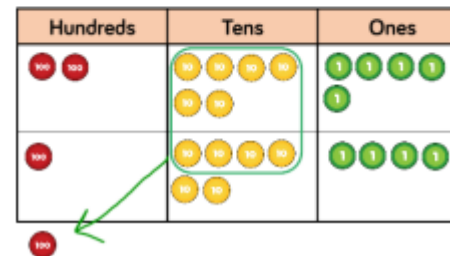
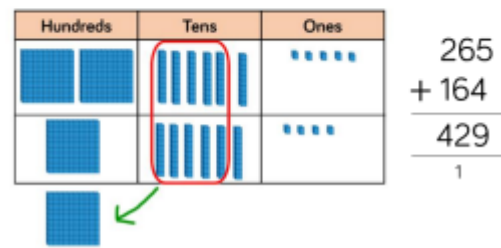
$$384$$

$$1$$

Ensure the children add the ones first and carry the numbers underneath.
Remind the pupils that the actual value is 40 + 30, not 4 + 3



$$265 + 164 = 429$$



Adding numbers with up to 4 digits

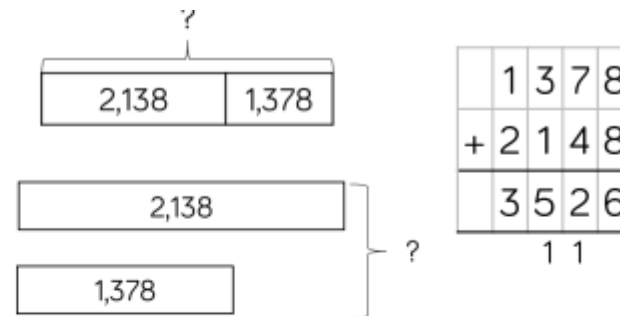
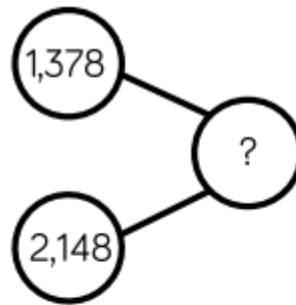
Base ten and place value counters are the most effective manipulatives when adding numbers with up to 3 digits. Ensure children write out their calculation alongside any concrete resources so they can see the link with the written column method. Plain counters on a place value grid can also be used to support learning.



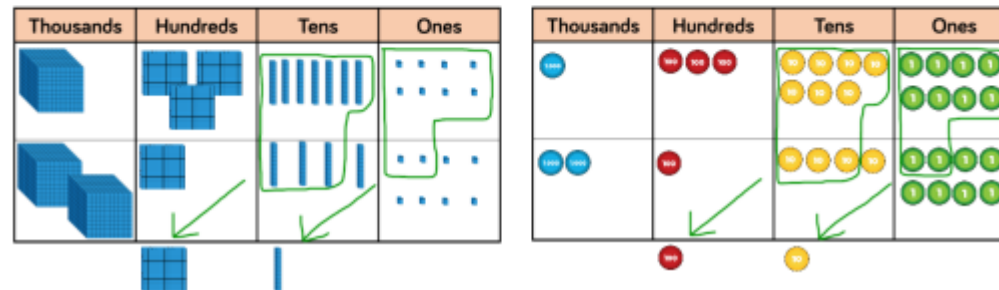
Adding numbers with more than 4 digits

$$3517 + 396$$

$$\begin{array}{r} 3517 \\ + 396 \\ \hline 3913 \\ \hline 11 \end{array}$$



$$1,378 + 2,148 = 3,526$$



Base ten and place value counters are the most effective manipulatives when adding numbers with up to 3 digits. Ensure children write out their calculation alongside any concrete resources so they can see the link with the written column method. Plain counters on a place value grid can also be used to support learning.



Add with numbers up to
3 decimal places

?

104,328 61,731

104,328

61,731

?

104,328 + 61,731 = 166,059

HTh	TTh	Th	H	T	O
100,000		1,000 1,000 1,000 1,000	300 100 100	30 30	1 1 1 1 1 1 1 1
	10,000 10,000 10,000 10,000 10,000 10,000	1,000	300 100 100 100 100 100 300	30 30 30	1

1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9
					1

Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits. At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.



Including money, measures and decimals with different numbers of decimal places

$$£23.59 + £7.55$$

$$\begin{array}{r} 23.59 \\ + 7.55 \\ \hline \end{array}$$

$$\begin{array}{r} \hline 111 \\ \hline \end{array}$$

$$19.01 + 3.65 + 0.7$$

$$\begin{array}{r} 19.01 \\ 3.65 \\ + 0.70 \\ \hline \end{array}$$

$$\begin{array}{r} \hline 23.36 \\ \hline \end{array}$$

$$\begin{array}{r} \hline 11 \\ \hline \end{array}$$

Pupils should be able to add more than two values, carefully aligning place value columns. Empty decimal places can be filled with a zero to show there is no value to add

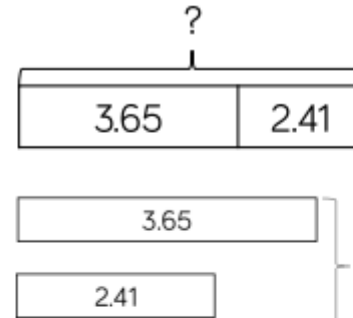
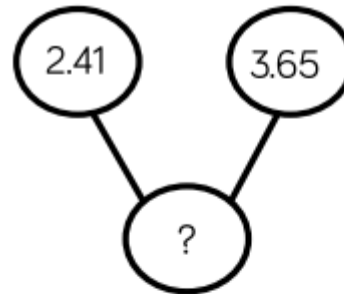
Adding several numbers with different numbers of decimal places. Tenths, hundredths and thousandths should be correctly aligned. Zeros can be added into empty decimal places, to show there is no value to add

$$23.361 + 9.08 + 59.77 + 1.3$$

$$\begin{array}{r} 23.361 \\ 9.080 \\ + 59.770 \\ \hline \end{array}$$



$$\begin{array}{r} 1.300 \\ 93.511 \\ \hline 212 \end{array}$$



$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}$$

$3.65 + 2.41 = 6.06$

Ones	Tenths	Hundredths
1 1 1	0.1 0.1 0.1 0.1 0.1 0.1	0.01 0.01 0.01 0.01 0.01
1 1	0.1 0.1 0.1 0.1	0.01

Ones	Tenths	Hundredths
••••	••••••••	•••••
••	•••••	•

Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1,2 and then 3 decimal places. Ensure children have experience of adding decimals with a variety of decimal places. This includes putting it into context when adding money and other measures.

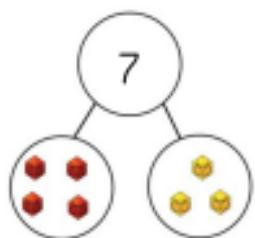
Benefits of Equipment, Models and Images



THURNBY MEAD
PRIMARY ACADEMY



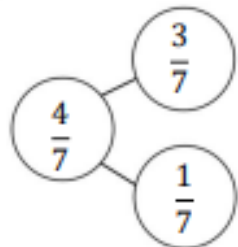
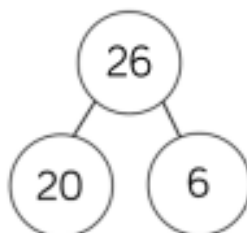
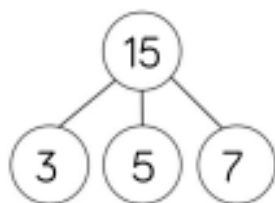
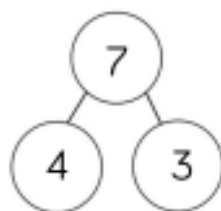
Part-Whole Model



$$7 = 4 + 3$$
$$7 = 3 + 4$$



$$7 - 3 = 4$$
$$7 - 4 = 3$$



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

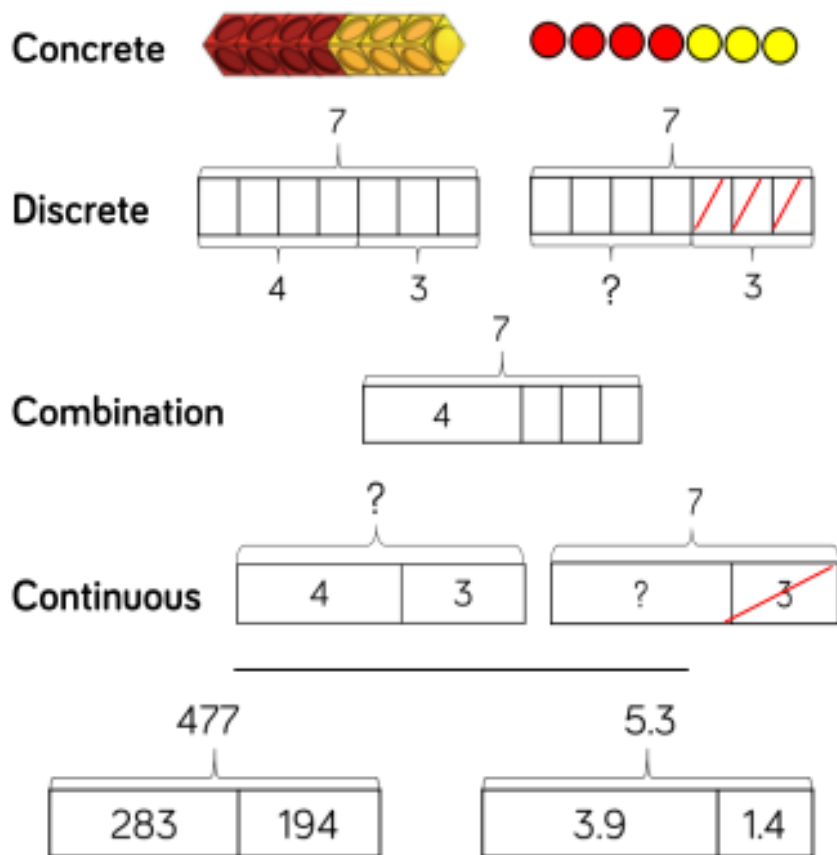
When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.



Bar Model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

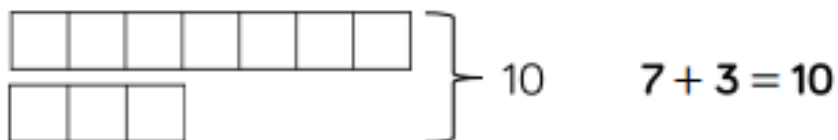
Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

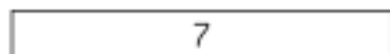


Bar Model (multiple)

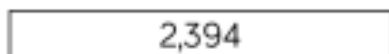
Discrete



Continuous



$$7 - 3 = 4$$



$$2,394 - 1,014 = 1,380$$

Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.



Number Shapes



$$7 = 4 + 3$$



$$7 = 3 + 4$$



$$7 - 3 = 4$$



$$6 + 4$$



$$7 + 3$$



$$8 + 2$$



$$9 + 1$$

Benefits

Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.



Cubes



$$7 = 4 + 3$$



$$7 = 3 + 4$$



$$7 - 3 = 4$$



$$7 - 3 = 4$$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

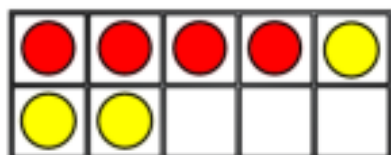
When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.



Ten Frames (within 10)



$$4 + 3 = 7$$

4 is a part.

$$3 + 4 = 7$$

3 is a part.

$$7 - 3 = 4$$

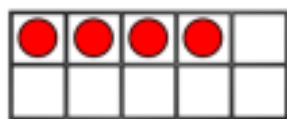
7 is the whole.

$$7 - 4 = 3$$

First



Then



$$4 + 3 = 7$$

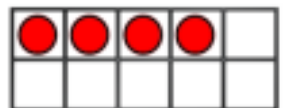
Now



First

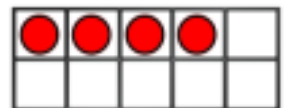


Then



$$7 - 3 = 4$$

Now



Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

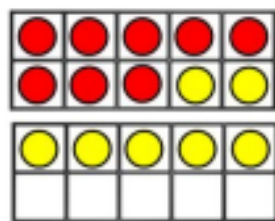
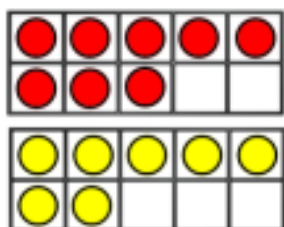
Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.

Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

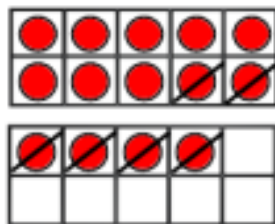
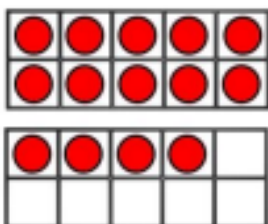
Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.



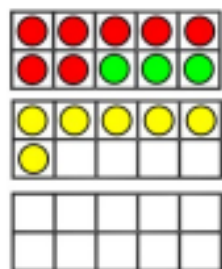
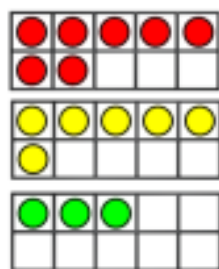
Ten Frames (within 20)



$$8 + 7 = 15$$



$$14 - 6 = 8$$



$$7 + 6 + 3 = 16$$

Benefits

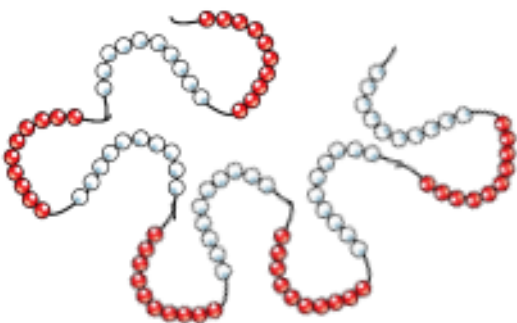
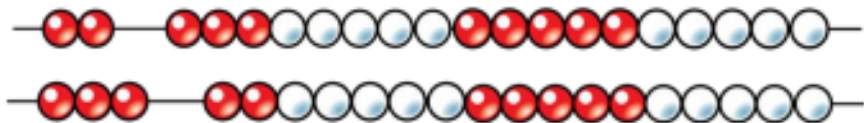
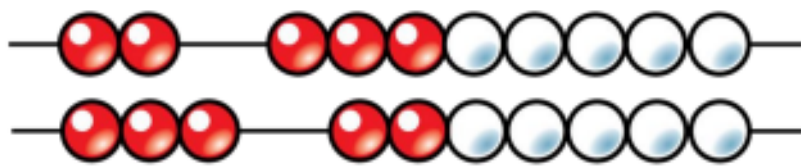
When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.



Bead Strings



Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10.

They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. $2 + 8 = 10$, move one bead, $3 + 7 = 10$.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

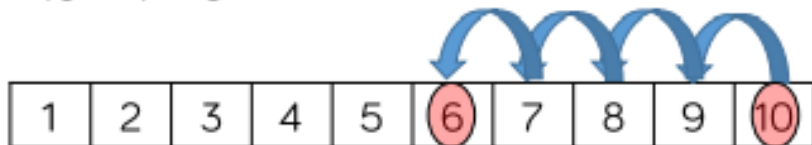


Number Tracks

$$5 + 3 = 8$$



$$10 - 4 = 6$$



$$8 + 7 = 15$$



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.



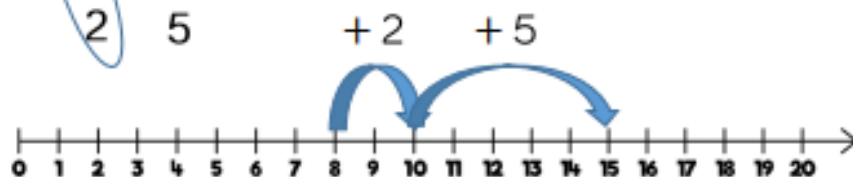
Number Lines (labelled)

$$5 + 3 = 8$$



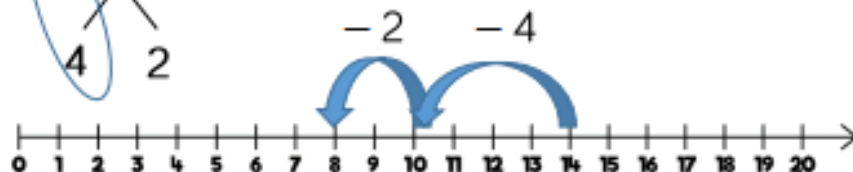
$$8 + 7 = 15$$

The number 8 is circled in blue. A bracket below it is split into two parts, labeled 2 and 5.



$$14 - 6 = 8$$

The number 14 is circled in blue. A bracket below it is split into two parts, labeled 4 and 2.



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

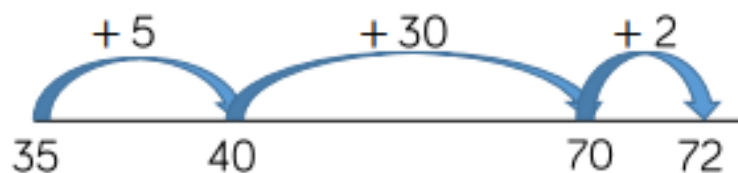
Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

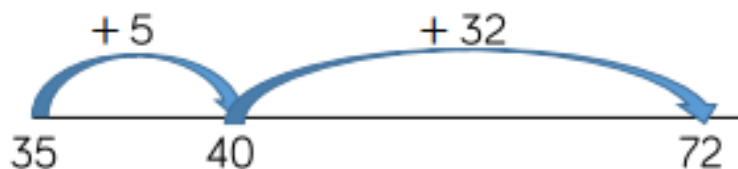


Number Lines (blank)

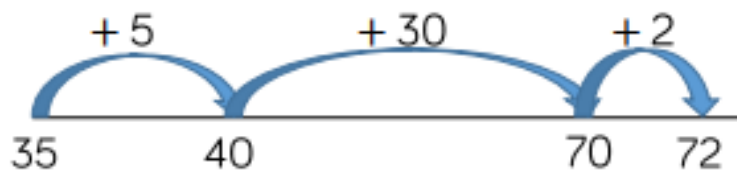
$$35 + 37 = 72$$



$$35 + 37 = 72$$



$$72 - 35 = 37$$



Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

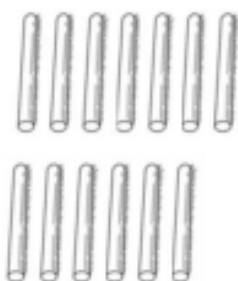
Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

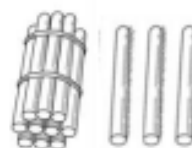


Straws

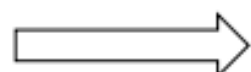
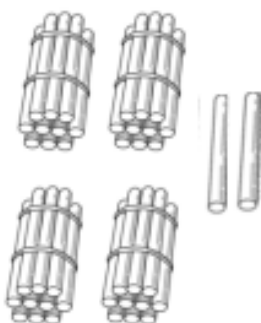
$$7 + 6 = 13$$



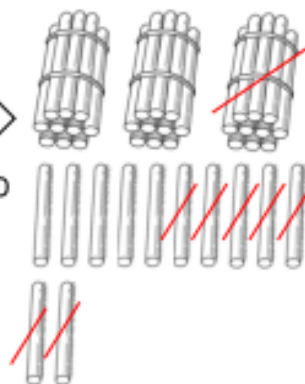
bundle together
groups of 10



$$42 - 17 = 25$$



unbundle group
of 10 straws



Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.



Base 10/Dienes (addition)

Tens	Ones

$$\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ 1 \end{array}$$

A blue vertical bar is positioned below the tens column of the model. A green arrow points from the circled 11 ones units in the model to the '1' written below the ones column in the calculation.

Hundreds	Tens	Ones

$$\begin{array}{r} 265 \\ + 164 \\ \hline 429 \\ 1 \end{array}$$

A blue hundred flat is positioned below the hundreds column of the model. A green arrow points from the circled 12 tens rods in the model to the '1' written below the tens column in the calculation.

Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.

- How many ones are there altogether?
- Can we make an exchange? (Yes or No)
- How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
- How many ones do we have left? (Write in ones column)

Repeat for each column.



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Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative - numbers can be added in any order.

Complement - in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference - the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange - Change a number or expression for another of an equal value.

Minuend - A quantity or number from which another is subtracted.

Partitioning - Splitting a number into its component parts.

Reduction - Subtraction as take away.

Subitise - Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total - The aggregate or the sum found by addition.