



Division as sharing
The equal sharing structure

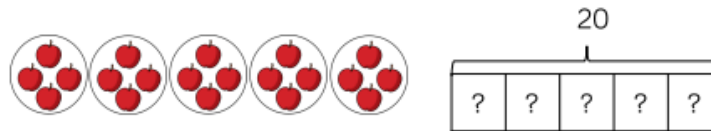
Introduce division as sharing (this requires the children to have secure counting skills and good one to one correspondence)

6 sweets are shared between 2 people. How many sweets will each person get?

The calculation corresponding to this situation is $6 \div 2$

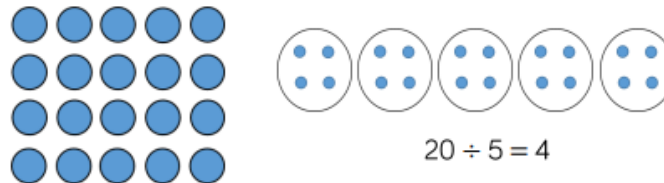


Practical activities involving sharing, distributing cards when playing a game, putting objects onto plates, into cups, hoops etc



Children in Year 1 are not expected to record division formally. In Year 2 children introduced to the division symbol

There are 20 apples altogether.
They are shared equally between 5 bags.
How many apples are in each bag?





Division as grouping
Inverse of multiplication
structure

Practical grouping e.g. in P.E
12 children get into teams of 4 to play a game. How many teams are there?
(As opposed to 12 children shared into 4 teams, how many children in each team?)

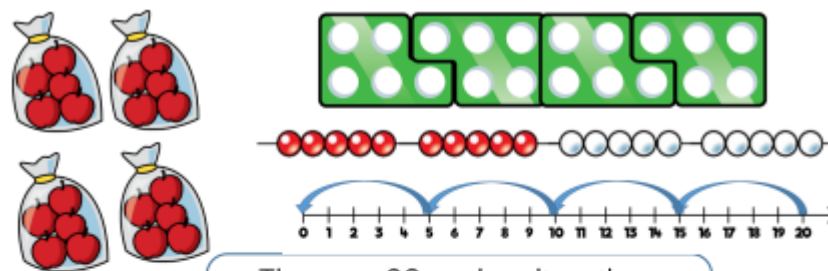


Sorting objects into 2s. How many pairs of socks are there?

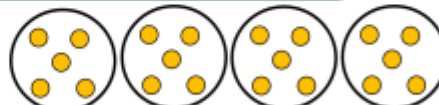
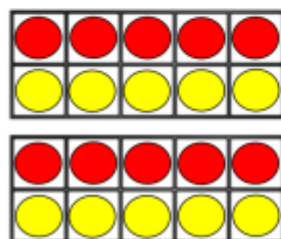


There are 6 strawberries. How many people can have 2 each? How many 2s make 6?

$6 \div 2 = 3$ 6 is the total and 2 is the group size



There are 20 apples altogether.
They are put in bags of 5.
How many bags are there?



$$20 \div 5 = 4$$

Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as Numicon which helps to show the link between multiplication and division.

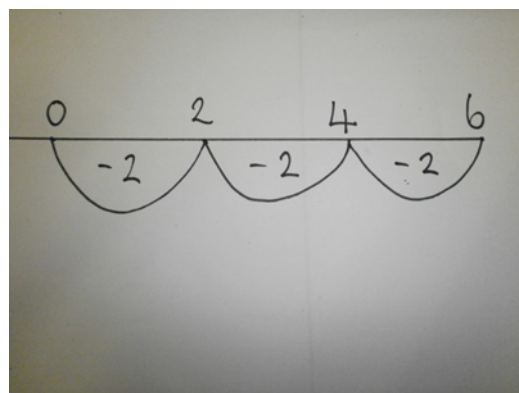
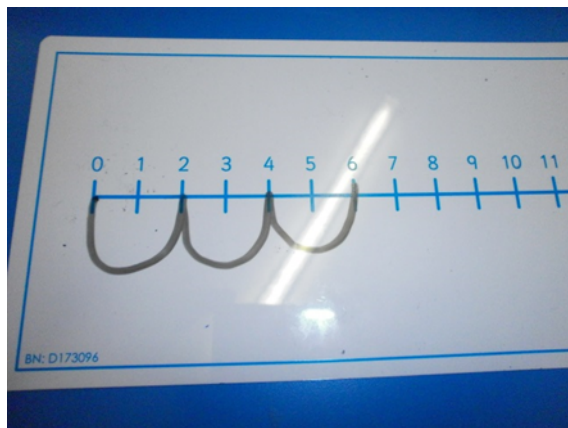


Understand division as repeated subtraction

$$6 \div 2 = 3$$

Have 6 objects and take away a group of 2 each time until you don't have any left. How many groups did you take away

Use a number line to demonstrate this as repeated subtraction



Draw own number line to show this, you can use equipment alongside this to show that you are taking away 2 each time



You can also show division as repeated addition:
How many groups of 2 do you need to make 6? Repeatedly adding 2 until you get to 6.

This can also be demonstrated using a bead string

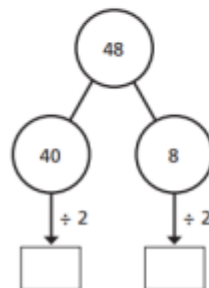
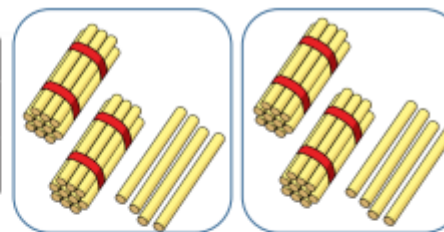
$$15 \div 3 = 5$$



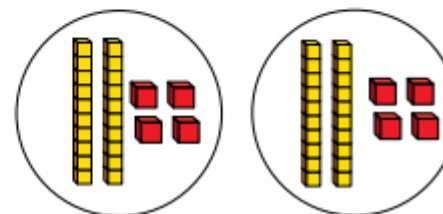


**Divide 2 digits by 1 digit
(sharing with no exchange)**

Tens		Ones			
10	10	1	1	1	1
10	10	1	1	1	1



$$48 \div 2 = 24$$



When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones. Straws, Base 10 and place value counters can all be used to share numbers into equal groups.



Division with remainders
The sharing structure

Introduce division with remainders practically

$$16 \div 3 = 5r1$$

Sharing – 16 shared between 3, how many are left over?



Grouping

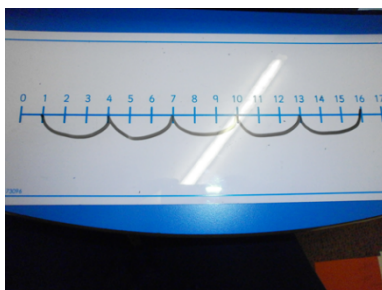
This is using the inverse of multiplication structure (grouping structure)

Starting with 16 objects consider how many groups of 3 you can make (demonstrate using biscuits, cakes, vegetables, fruit etc and put them into bags of 3). How many complete packets?



How many bags of fruit will I need to buy for 16 people? Five won't be enough

Introduce division with remainders on a printed number and then use own number line



With some division calculations we need to round up or round down when giving the answer

Round up

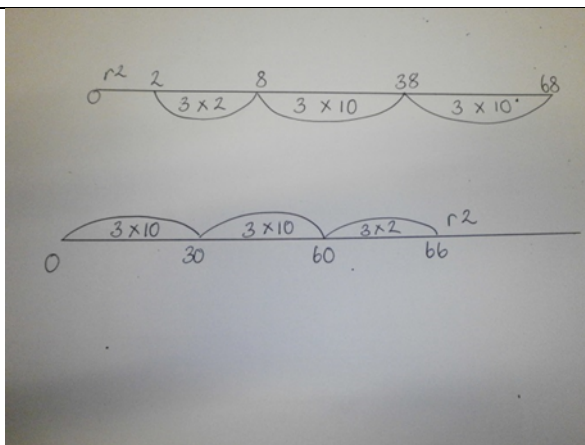
How many buses are needed to take 246 people to a one direction concert when each bus holds 10 people?

Round down

Eggs come in boxes of 6. How many full boxes will there be if you had 70 eggs?



Introduce division with and without remainders – chunking on a number line



$$85 \div 5$$

Using knowledge of the 5 times table to count back in efficient chunks. 5×10 , 5×5 and 5×1 are useful ones to know if the children can't recall all of the times table.

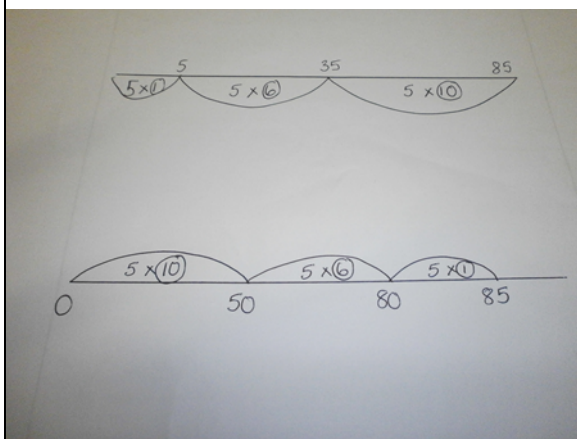
Distributive laws of division means that the dividend can be split into $(80 + 5) \div 5$

$$\text{Or } (a+b) \div c$$

$$\text{Or } (a - b) \div c$$

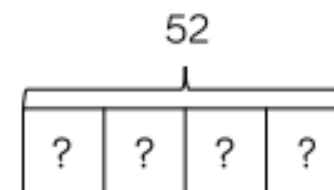
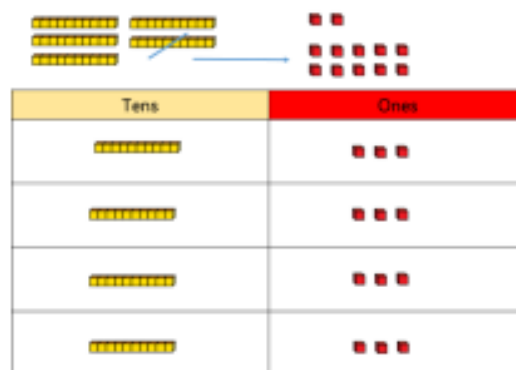
$$\text{Or } 50 + 30 + 5$$

$$68 \div 3$$

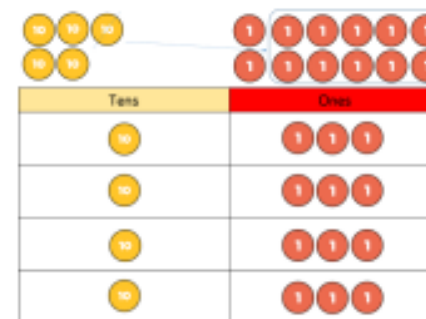
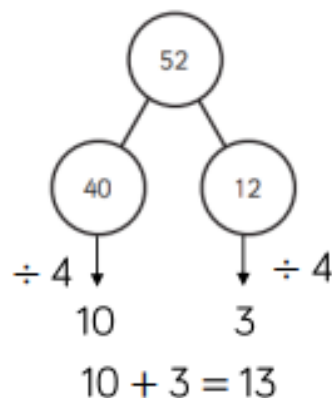




Divide 2-digit numbers by
1-digit numbers (sharing
with exchange)



$$52 \div 4 = 13$$



When dividing numbers involving an exchange, children can use Base-ten and place value counters to exchange one ten for ten ones. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows. Flexible partitioning () in a part whole model helps with this method.



Divide 2 digits by 1 digit
(sharing with remainders)

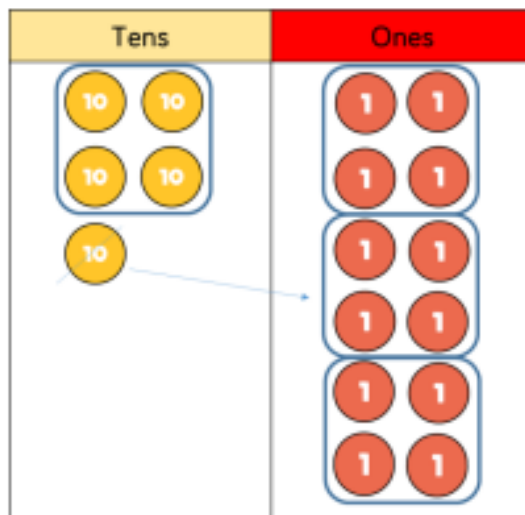
The diagrams illustrate the division of 53 by 4 using various methods:

- Base 10 Grid:** A grid with 5 rows and 2 columns. The left column is labeled 'Tens' and contains 5 yellow bars. The right column is labeled 'Ones' and contains 3 red dots. An arrow points from the top-left corner to a separate group of 4 red dots, representing the remainder.
- Part-Whole Model:** A tree diagram starting with 53 at the top. It branches into 40 and 13. 40 branches into 12 and 1. 12 branches into 8 and 4. 8 branches into 4 and 4. 4 branches into 2 and 2. 2 branches into 1 and 1. The final result is 13 and 1.
- Number Line:** A number line starting at 0 and ending at 53. It is marked with 0, 10, 20, 30, 40, and 53. There are 13 intervals of 4 units each, with a final interval of 1 unit.
- Equation:** $53 \div 4 = 13 \text{ r}1$
- Base 10 Grid with Remainder:** A grid with 5 rows and 2 columns. The left column is labeled 'Tens' and contains 5 yellow bars. The right column is labeled 'Ones' and contains 13 red dots. A single red dot is placed to the right of the grid, representing the remainder.

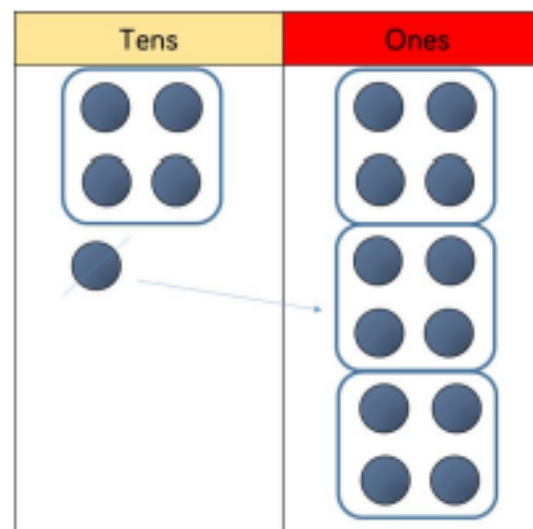
When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones. Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made. Flexible partitioning () in a part-whole model supports this method.



Divide 2 digits by 1-digit
(grouping)



$$52 \div 4 = 13$$

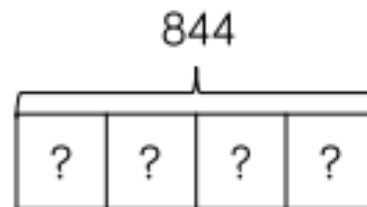


When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor. Language is important here. Children should consider 'How many groups of 10 can we make?' and 'How many groups of four ones can we make?' Remainders can also be seen as they remain ungrouped.

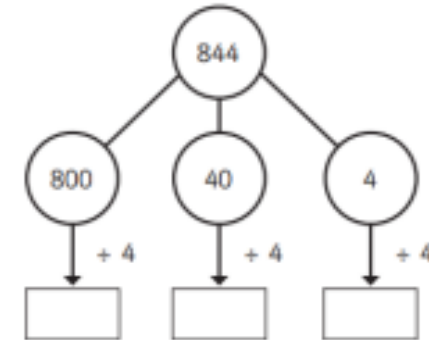


Divide 3 digits by 1-digit
(sharing)

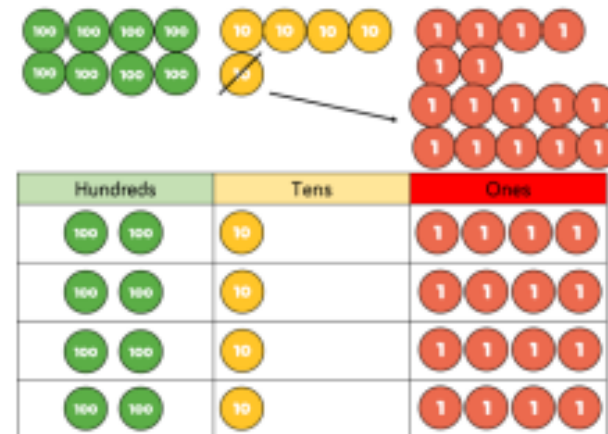
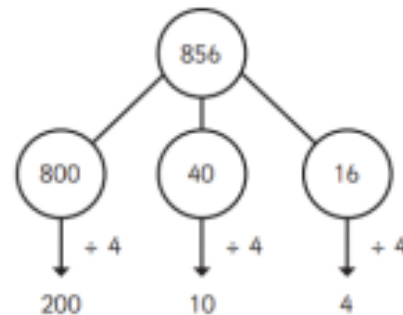
$$844 \div 4 = 211$$



H	T	O
100 100	10	1
100 100	10	1
100 100	10	1
100 100	10	1



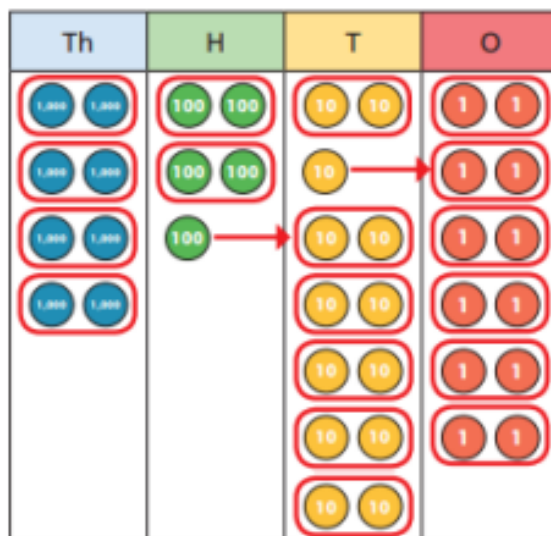
$$844 \div 4 = 211$$





**Divide 4 digits by 1 digit
(grouping)**

Children can continue to use place value counters to share 3-digit numbers into equal groups. Children should start outside the place value grid before sharing the hundreds, tens, and ones equally between the rows. This method can also help to highlight remainders. Flexible partitioning in a part whole method can support this model.



	4	2	6	6
2	8	5	13	12

$$8,532 \div 2 = 4,266$$

Place value counters or plain counters can be used on a place value grid to support children to divide 4 digits by 1 digit. Children can also draw their own counters and group them in a more pictorial method. Children should be encouraged to move away from the concrete and the pictorial when dividing numbers with multiple exchanges.



Divide multi digits by 2
digits (short division)

		0	3	6
	12	4	⁴ 3	⁷ 2

$$432 \div 12 = 36$$

$$7,335 \div 15 = 489$$

	0	4	8	9
15	7	⁷ 3	¹³ 3	¹³ 5

15	30	45	60	75	90	105	120	135	150
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Divide multi digits by 2-digits (long division- to be used to support children's understanding and as an additional method if required).

When children begin to divide up to 4 digits by 2 digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

		0	3	6
1	2	4	3	2
	-	3	6	0
			7	2
	-		7	2
				0

(x30)

(x6)

- 12 × 1 = 12
- 12 × 2 = 24
- 12 × 3 = 36
- 12 × 4 = 48
- 12 × 5 = 60
- 12 × 6 = 72
- 12 × 7 = 84
- 12 × 8 = 96
- 12 × 9 = 108
- 12 × 10 = 120

432 ÷ 12 = 36

7,335 ÷ 15 = 489

		0	4	8	9
15	7	3	3	5	
-	6	0	0	0	
	1	3	3	5	
-	1	2	0	0	
		1	3	5	
-		1	3	5	
				0	

(x400)

(x80)

(x9)

- 1 × 15 = 15
- 2 × 15 = 30
- 3 × 15 = 45
- 4 × 15 = 60
- 5 × 15 = 75
- 10 × 15 = 150



Children can also divide by 2-digit numbers by long division. Children can write out multiples to support their calculations using larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Divide multi-digits by 2-digits (long division- to be used to support children's understanding and as an additional method if required).

$$372 \div 15 = 24 \text{ r}12$$

			2	4	r	1	2
1	5	3	7	2			
-		3	0	0			
			7	2			
-			6	0			
			1	2			

- 1 × 15 = 15
- 2 × 15 = 30
- 3 × 15 = 45
- 4 × 15 = 60
- 5 × 15 = 75
- 10 × 15 = 150

			2	4	$\frac{4}{5}$
1	5	3	7	2	
-		3	0	0	
			7	2	
-			6	0	
			1	2	

$$372 \div 15 = 24 \frac{4}{5}$$



When a remainder is left at the end of the calculation, children can either leave it as a remainder or convert it to a fraction. This will depend upon the context of the question. Children can also answer questions where the quotient needs to be rounded according to the context.

Short division
 $98 \div 7$ becomes

$$\begin{array}{r} 14 \\ 7 \overline{) 98} \\ \underline{7} \\ 28 \\ \underline{28} \\ 0 \end{array}$$

Answer: 14

$432 \div 5$ becomes

$$\begin{array}{r} 86 \text{ r}2 \\ 5 \overline{) 432} \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$$

Answer: 86 remainder 2

$496 \div 11$ becomes

$$\begin{array}{r} 45 \text{ r}1 \\ 11 \overline{) 496} \\ \underline{44} \\ 56 \\ \underline{55} \\ 1 \end{array}$$

Answer: $45 \frac{1}{11}$

Long division
 $432 \div 15$ becomes

$$\begin{array}{r} 28 \text{ r}12 \\ 15 \overline{) 432} \\ \underline{30} \\ 132 \\ \underline{120} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

Answer: 28 remainder 12

$432 \div 15$ becomes

$$\begin{array}{r} 28 \\ 15 \overline{) 432} \\ \underline{30} \\ 132 \\ \underline{120} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

$\frac{12}{15} = \frac{4}{5}$

Answer: $28 \frac{4}{5}$

$432 \div 15$ becomes

$$\begin{array}{r} 28.8 \\ 15 \overline{) 432.0} \\ \underline{30} \\ 132 \\ \underline{120} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

Answer: 28.8

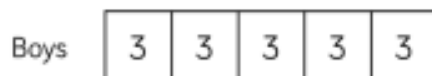
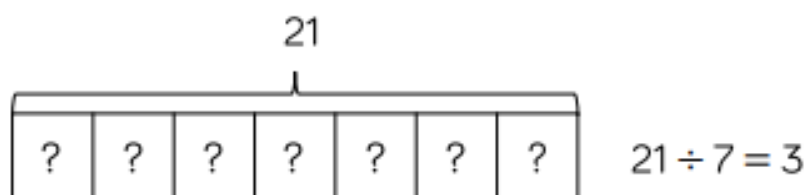
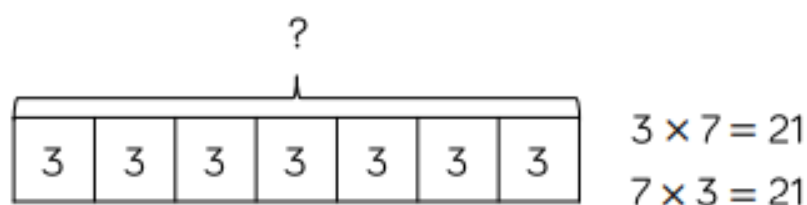
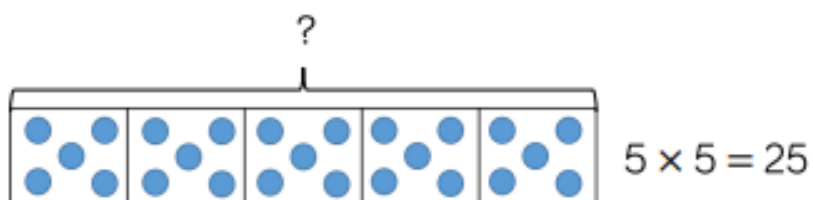


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Bar Model



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?

The multiple bar model provides an opportunity to compare the groups.



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Number Shapes



$$5 \times 4 = 20$$

$$4 \times 5 = 20$$

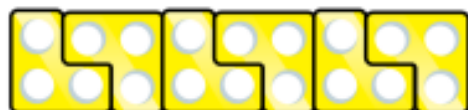


$$5 \times 4 = 20$$

$$4 \times 5 = 20$$



$$18 \div 3 = 6$$



Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd \times odd = even, odd \times even = odd, even \times even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.



Bead Strings



$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

$$15 \div 3 = 5$$



$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

$$15 \div 5 = 3$$



$$4 \times 5 = 20$$

$$5 \times 4 = 20$$

$$20 \div 4 = 5$$

Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.

Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 - Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.



Number Tracks



$$6 \times 3 = 18$$

$$3 \times 6 = 18$$



$$18 \div 3 = 6$$

Benefits

Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.

When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

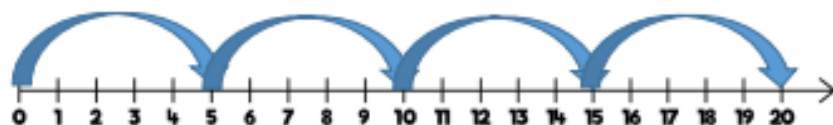
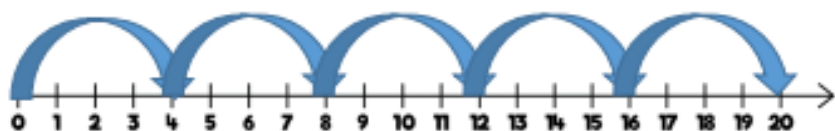
Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.



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Number Lines (labelled)



$$4 \times 5 = 20$$

$$5 \times 4 = 20$$



$$20 \div 4 = 5$$

Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.

When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.

Children record how many jumps they have made to find the answer to the division.

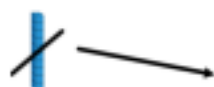
Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.



Base 10/Dienes (division)

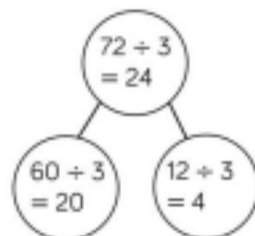


$$68 \div 2 = 34$$



Tens	Ones

$$72 \div 3 = 24$$



Benefits

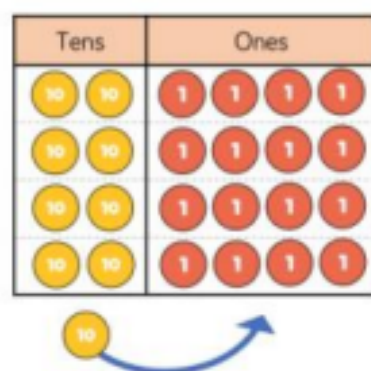
Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

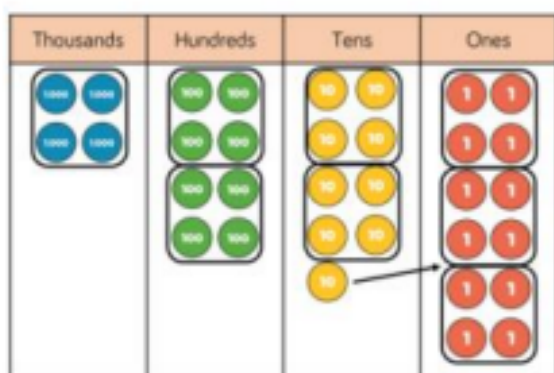
When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.



Place Value Counters (division)



$$\begin{array}{c} 96 \div 4 \\ = 24 \\ \swarrow \quad \searrow \\ 80 \div 4 = 20 \quad 16 \div 4 = 4 \end{array}$$



$$\begin{array}{r} 1223 \\ 4 \overline{) 4892} \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.



Glossary

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor



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