



Calculation Policy

Multiplication

Progression of Skills	How to do it
Multiplication	Multiplication has 2 structures: <ul style="list-style-type: none">• The repeated aggregation structure or repeated addition• The scaling structure associated with scale models and scale drawings
Arrays	Know the difference between fair and unfair
Repeated addition	Play with commercial arrays (bun trays, egg boxes, yogurt containers, paint trays). Represent numbers in arrays in the playground
Counting in steps	Listen to stories about multiplication Sing songs and listen to rhymes Act out counting out equal sets of objects e.g. socks Recognise that there are more objects as a result of doubling Record activities using objects, pictures, diagrams and mark making Give children routine tasks e.g. put 3 sweets on each plate for our teddies. Model the language Groups of ____ on each plate
Doubling	Count in twos fives and tens: Use fingers to show counting five at a time and two hands to show counting 10 at a time Socks and shoes to show counting in 2s Repeated Numicon shapes
	Use practical and visual resources to show doubling. Use vocabulary of double and twice as many (the same again) Build towers Repeated Numicon shapes Counting groups of objects and making another group with the same amount.

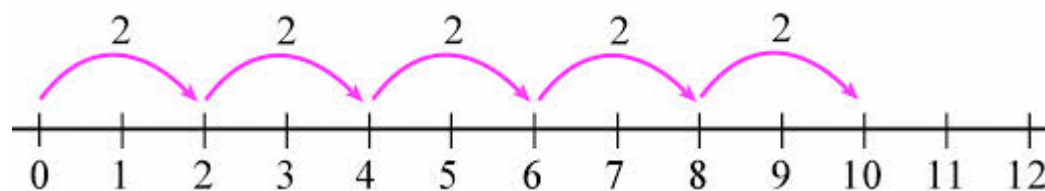


**Understand
multiplication as
repeated addition**

Multiplication is related to doubling and counting groups of the same size



Use a washing line and other practical resources for counting. Concrete objects, Numicon, bundles of straws, bead strings etc



$$2 + 2 + 2 + 2 + 2 = 10$$

$$2 \times 5 = 10$$

$$5 \times 2 = 10$$

2 multiplied by 5



5 multiplied by 2
5 pairs
5 jumps of 2

$$5 + 5 + 5 = 15$$

$$5 \times 3$$

$$3 \times 5$$

5 multiplied by 3

3 multiplied by 5

5 three times

3 groups of 5



Problem solving with concrete objects (including money and measures)

There are 2 sweets in a bag, how many sweets are there in 3 bags



$$2 + 2 + 2 = 6$$

$$2 \times 3 = 6$$

$$3 \times 2 = 6$$

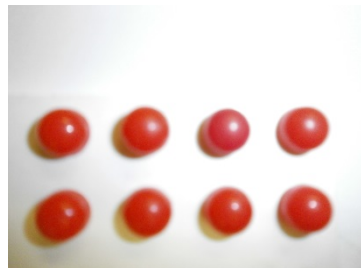
2 multiplied by 3

2 three times

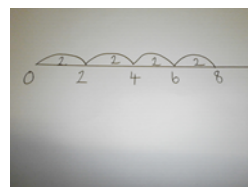
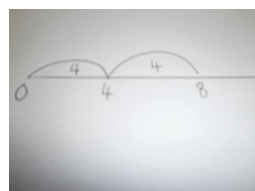
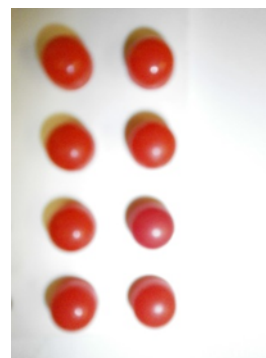


Understand multiplication as an array

Use arrays to understand multiplication



$$4 \times 2$$
$$4 + 4$$



$$2 \times 4$$
$$2 + 2 + 2 + 2$$

Use arrays to understand that multiplication can be done in any order – it is commutative

Express multiplication as a number sentence

Express multiplication as a number sentence using x

Using understanding of the inverse and practical resources solve missing number problems

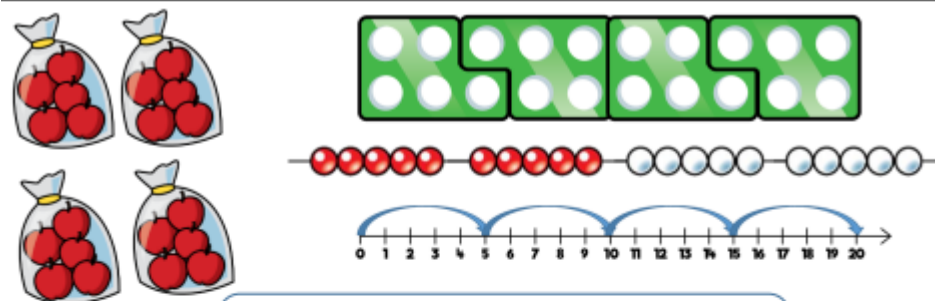
$$7 \times 2 = ? \quad ? = 2 \times 7$$

$$7 \times ? = 14 \quad 14 = ? \times 7$$

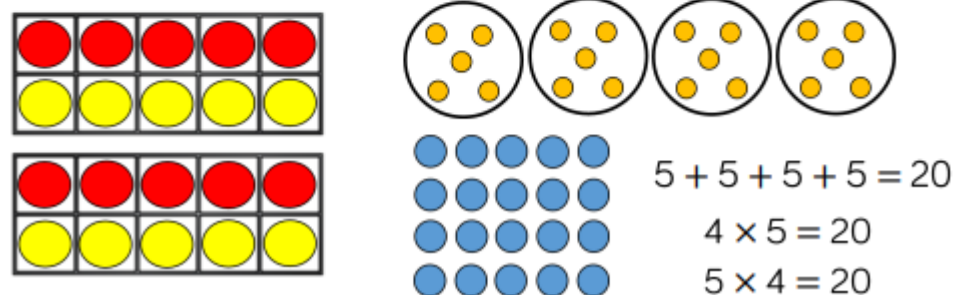
$$? \times 2 = 14 \quad 14 = 2 \times ?$$

$$? \times ? = 14 \quad 14 = ? \times ?$$

Develop understanding of multiplication using arrays and number lines. Include multiplications not in the 2, 5 and 10 times table



One bag holds 5 apples.
How many apples do 4 bags hold?



In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally. In Year 2 children are introduced to the multiplication symbol.

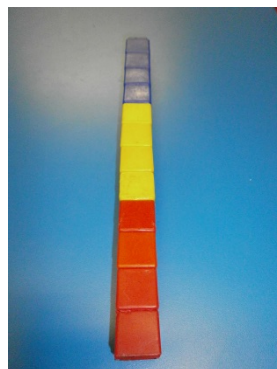


Understand multiplication as scaling

Begin to develop an understanding of scaling (3 times bigger/taller)

$$4 \times 3$$

$$3 \times 4$$



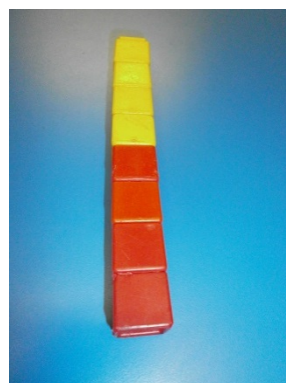
$$4 \times 3$$

There are 4 cubes in a tower. If I had 3 times as many cubes how many would I have?

Double numbers

Doubling numbers to 10 + 10

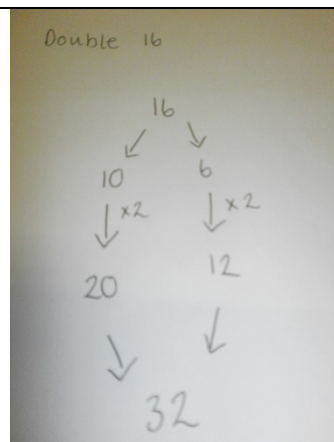
Link language of doubling with the language of twice as many



Double 4 is 8

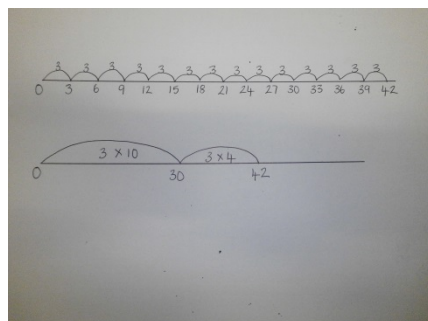
$$4 \times 2 = 8$$

Use jottings to develop an understanding of doubling 2 digit numbers



Demonstrate multiplication on a number line jumping in larger steps

3×14

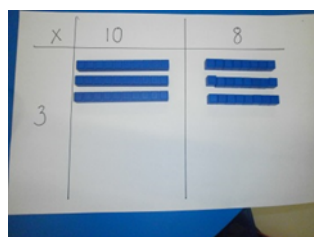


Develop written methods with an understanding of visual images

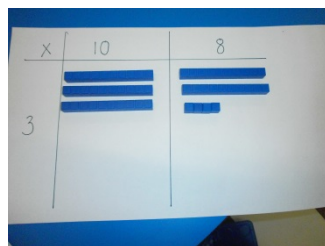
18×3



Link arrays to the practical grid method



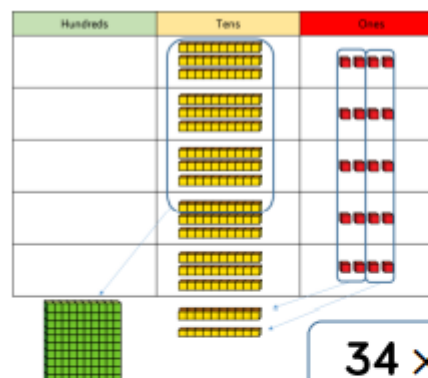
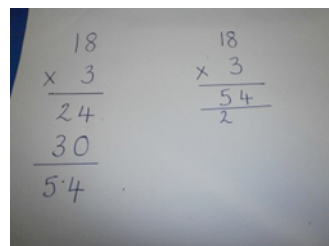
Next, ask them to regroup the ones into tens and ones and to show the answer linking it to the way in which the children would solve the calculation



Using these models, demonstrate how the array can be transferred to the partitioning method for multiplication and then progress to the more formal method using columns

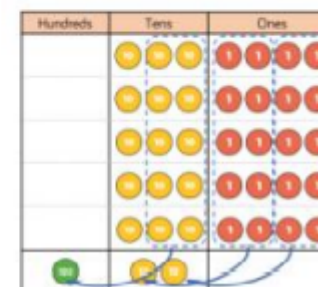


Multiply 2-digit numbers by 1-digit numbers



	H	T	O	
		3	4	
x			5	
		2	0	(5 x 4)
+	1	5	0	(5 x 30)
	1	7	0	

	H	T	O	
		3	4	
x			5	
	1	7	0	
	1	2		



You may decide to look at the grid method, then the expanded column method before moving on to the short multiplication method. The place value counters should be used to support the understanding of the method rather than supporting the calculation, as children should use their times table knowledge.



Multiply 3-digit numbers by 1-digit numbers

The diagram illustrates the multiplication of 245 by 4 using place value charts and base 10 blocks.

Place Value Chart (Base 10 Blocks):

Hundreds	Tens	Ones
200 (2 green squares)	40 (4 yellow rods)	5 (5 red units)
200 (2 green squares)	40 (4 yellow rods)	5 (5 red units)
200 (2 green squares)	40 (4 yellow rods)	5 (5 red units)
200 (2 green squares)	40 (4 yellow rods)	5 (5 red units)

Formal Written Method:

	H	T	O
	2	4	5
x			4
<hr/>			
	9	8	0
	1	2	

Equation: $245 \times 4 = 980$

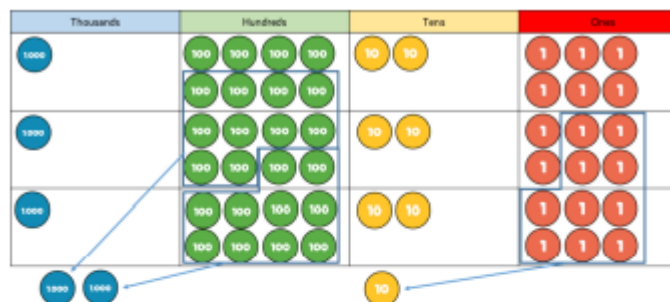
Place Value Chart (Base 10 Counters):

Hundreds	Tens	Ones
200 (2 green circles)	40 (4 yellow rods)	5 (5 red units)
200 (2 green circles)	40 (4 yellow rods)	5 (5 red units)
200 (2 green circles)	40 (4 yellow rods)	5 (5 red units)
200 (2 green circles)	40 (4 yellow rods)	5 (5 red units)

When moving to 3-digit by 1-digit multiplication, encourage the children to move towards the short, formal written method. Base 10 and place value counters continue to support the understanding of the written method. Limit the number of exchanges needed in the questions and move children away from the resources when multiplying larger numbers.



Multiply 4-digit numbers by 1-digit numbers



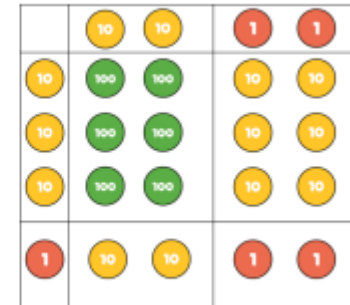
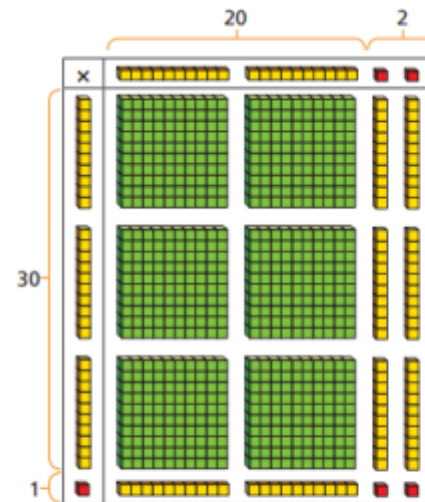
$$1,826 \times 3 = 5,478$$

	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8
	2		1	

When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method. If children are multiplying larger numbers and are struggling with their tables, encourage the use of multiplication grids so children can focus on the use of the written method.



Multiply 2-digit numbers by 2-digit numbers



×	20	2
30	600	60
1	20	2

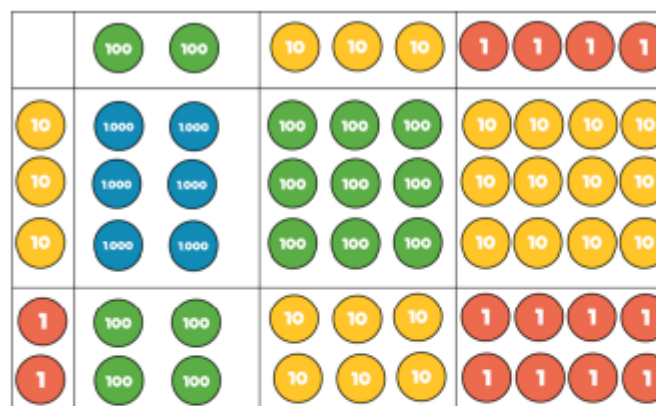
	H	T	O
		2	2
×		3	1
<hr/>			
	6	6	0
	6	8	2

$$22 \times 31 = 682$$

When multiplying a multi-digit number by 2 digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by base 10. The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.



Multiply 3-digit numbers by 2-digit numbers



	Th	H	T	O
		2	3	4
x			3	2
		4	6	8
1	7	1	0	2
	7	4	8	8

$$234 \times 32 = 7,488$$

x	200	30	4
30	6,000	900	120
2	400	60	8

Children can continue the use of the area model when multiplying 3 digits by 2 digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of the numbers. Encourage the children to move towards the formal written method, seeing the link with the grid method.



Linking grid method to formal algorithm (if appropriate and to support children's understanding).

Skills needed: Rounding to the nearest 10 or 100 to estimate calculation
Partitioning numbers into 100s 10s and 1s
Quick recall of multiplication facts
Ability to multiply by 10 or 100
Efficient addition strategy to sum the products

834 x 8 is approximately 800 x 8 = 6400

x	800	30	4
8	6400	240	32
			= 6672

72 x 38 is approximately 70 x 40 = 2800



x	70	2	
30	2100	60	= 2160
8	560	16	576
			<u>2736</u>

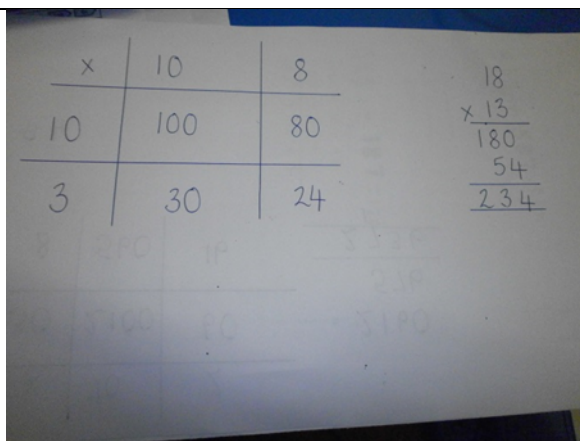
Summing the products can be carried out using a mental or written method

9.4 x 6 is approximately 9 x 6 = 54

x	9	0.4	
6	54	2.4	= 56.4

18 x 13

Children to explore how the grid method supports an understanding of long multiplication



Once the children have an understanding of this the expectation would be to do it without writing each calculation



Multiply 4-digit numbers by 2-digit numbers

TTh	Th	H	T	O
	2	7	3	9
×			2	8
<hr/>				
2	1	9	1	2
₂	₅	₃	₇	
<hr/>				
5	4	7	8	0
₁		₁		
<hr/>				
7	6	6	9	2
		₁		

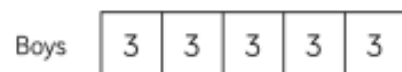
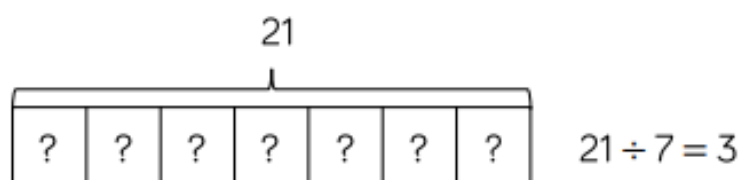
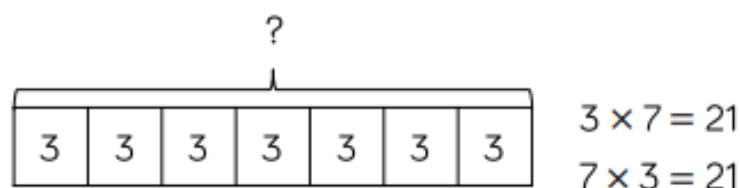
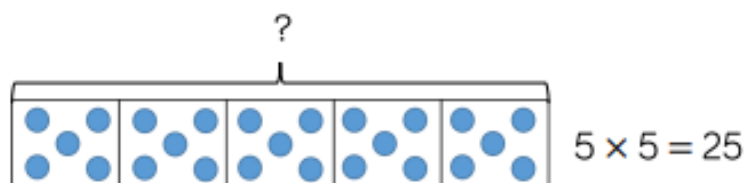
$$2,739 \times 28 = 76,692$$

When multiplying 4-digits by 2 digits, children should be confident in the written method. If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method.



Benefits of Equipment, Models and Images

Bar Model



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?

The multiple bar model provides an opportunity to compare the groups.



Number Shapes



$$5 \times 4 = 20$$

$$4 \times 5 = 20$$

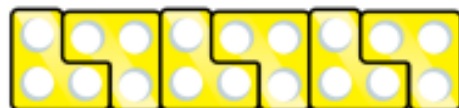


$$5 \times 4 = 20$$

$$4 \times 5 = 20$$



$$18 \div 3 = 6$$



Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd \times odd = even, odd \times even = odd, even \times even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.



Bead Strings



$$5 \times 3 = 15$$
$$3 \times 5 = 15$$

$$15 \div 3 = 5$$



$$5 \times 3 = 15$$
$$3 \times 5 = 15$$

$$15 \div 5 = 3$$



$$4 \times 5 = 20$$
$$5 \times 4 = 20$$

$$20 \div 4 = 5$$

Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.

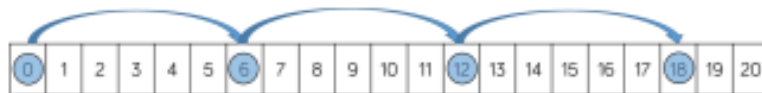
Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.



Number Tracks



$$6 \times 3 = 18$$

$$3 \times 6 = 18$$



$$18 \div 3 = 6$$

Benefits

Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

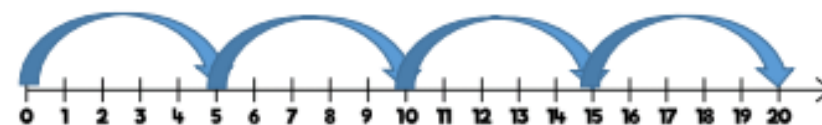
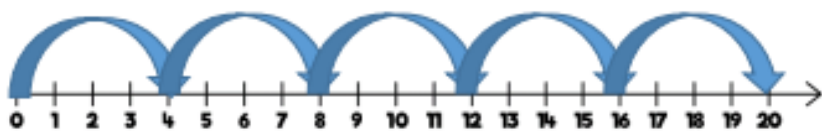
When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.

When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

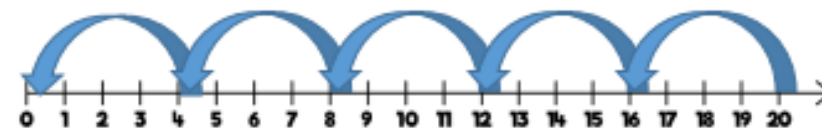


Number Lines (labelled)



$$4 \times 5 = 20$$

$$5 \times 4 = 20$$



$$20 \div 4 = 5$$

Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.

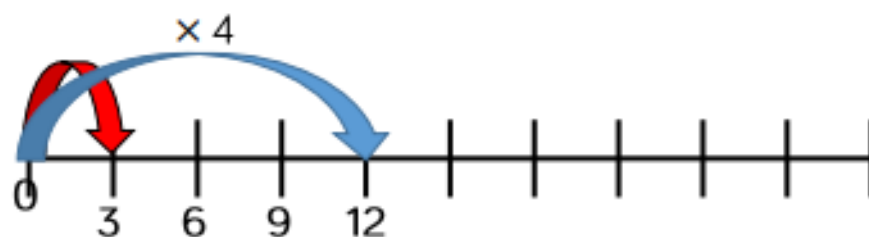
When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.

Children record how many jumps they have made to find the answer to the division.

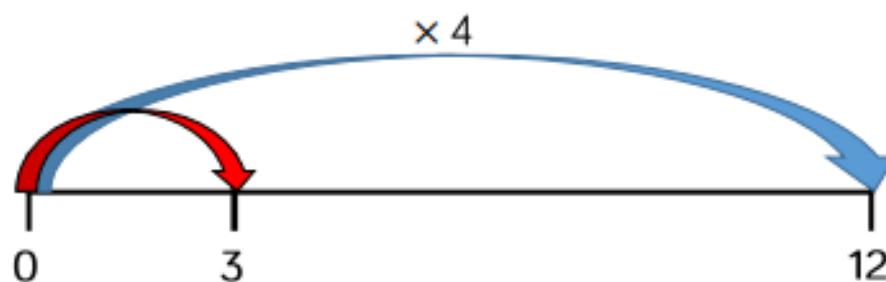
Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.



Number Lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

Benefits

Children can use blank number lines to represent scaling as multiplication or division.

Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

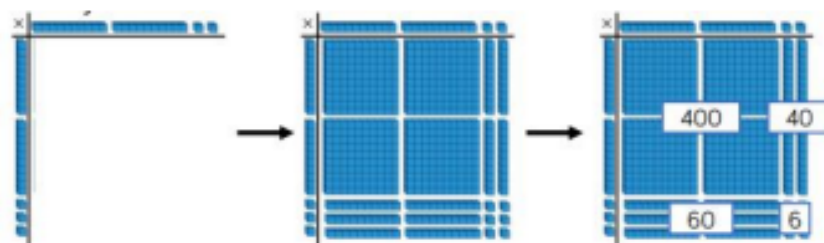


Base 10/Dienes (multiplication)

Hundreds	Tens	Ones
		●●●●
		●●●●
		●●●●

←

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ \hline 1 \end{array}$$



Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces. This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.



Place Value Counters (multiplication)

Hundreds	Tens	Ones
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
100	10 10	

$$\begin{array}{r} 34 \\ \times 5 \\ \hline 170 \\ 12 \end{array}$$

x	10 10 10 10	1 1 1 1
10	100 100 100 100	10 10 10 10
10	100 100 100 100	10 10 10 10
10	100 100 100 100	10 10 10 10
1	10 10 10 10	1 1 1 1
1	10 10 10 10	1 1 1 1

$$\begin{array}{r} 44 \\ \times 32 \\ \hline 8 \\ 80 \\ + 120 \\ \hline 1408 \\ 1 \end{array}$$

Benefits


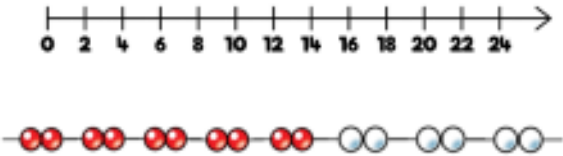



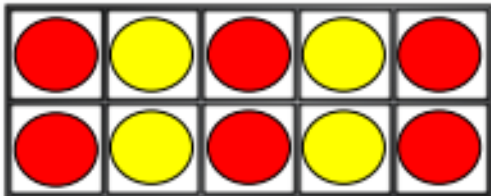
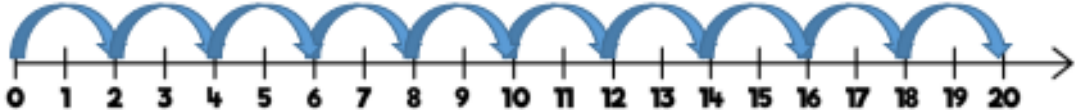
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed. The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.



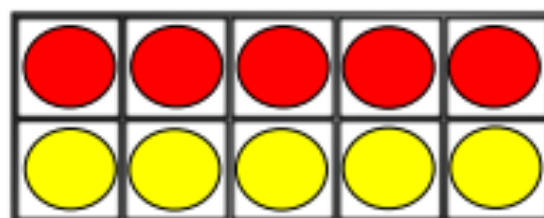
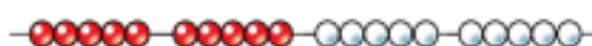
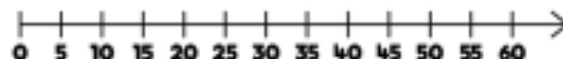
Times Tables

Skill: 2 times table	Year: 2
 	<p>Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.</p>
 	<p>Look for patterns in the two times table, using concrete manipulatives to support. Notice how all the numbers are even and there is a pattern in the ones.</p>
 	
	<p>Use different models to develop fluency.</p>

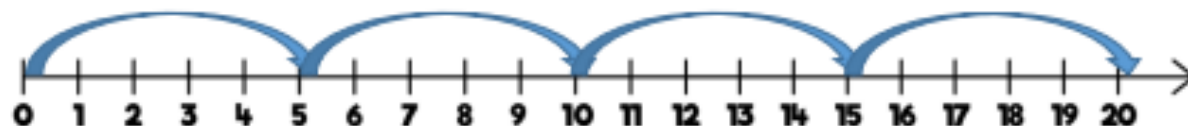
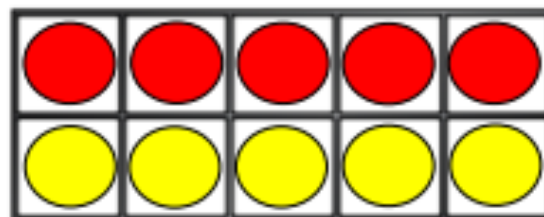


Skill: 5 times table

Year: 2



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



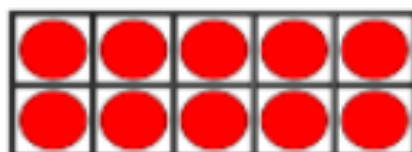
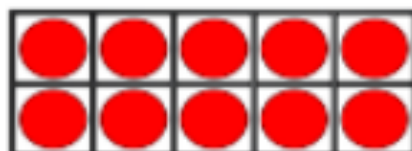
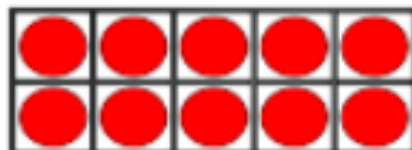
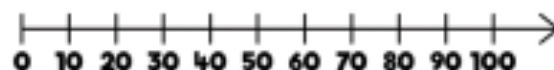
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the five times table, using concrete manipulatives to support. Notice the pattern in the ones as well as highlighting the odd, even, odd, even pattern.



Skill: 10 times table

Year: 2



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

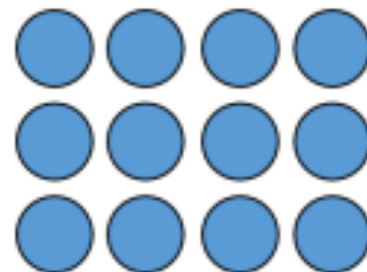
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the ten times table, using concrete manipulatives to support. Notice the pattern in the digits- the ones are always 0, and the tens increase by 1 ten each time.

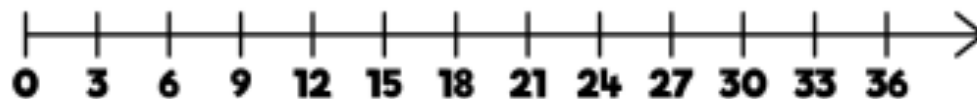
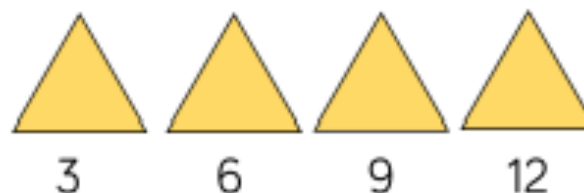


Skill: 3 times table

Year: 3



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the three times table, using concrete manipulatives to support. Notice the odd, even, odd, even pattern using number shapes to support. Highlight the pattern in the ones using a hundred square.



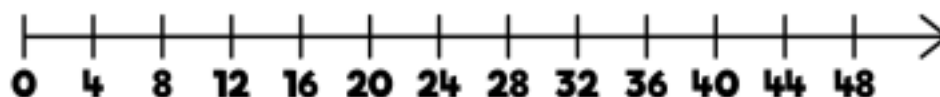
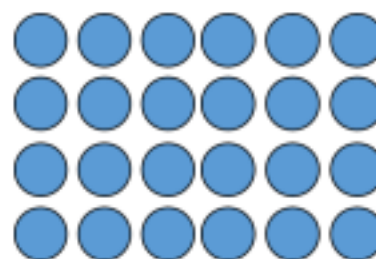
Skill: 4 times table

Year: 3



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

4	8	12	16	20
24	28	32	36	40
44	48	52	56	60



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the four times table, using manipulatives to support. Make links to the 2 times table, seeing how each multiple is double the twos. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.



Skill: 8 times table

Year: 3



8

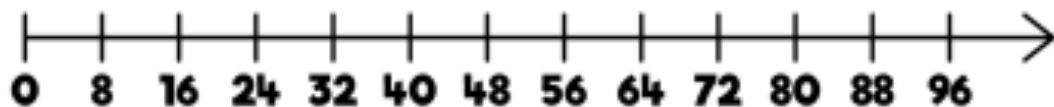
16

24

32

8	16	24	32	40
48	56	64	72	80

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the eight times table, using manipulatives to support. Make links to the 4 times table, seeing how each multiple is double the fours. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.



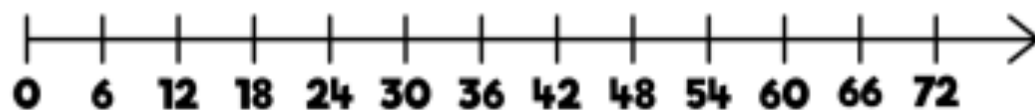
Skill: 6 times table

Year: 4



6	12	18	24	30
36	42	48	54	60
66	72	78	84	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the six times table, using manipulatives to support. Make links to the 3 times table, seeing how each multiple is double the threes. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.



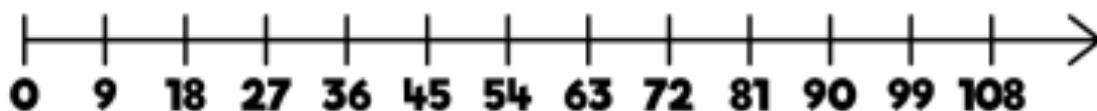
Skill: 9 times table

Year: 4



9	18	27	36	45
54	63	72	81	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the nine times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support as well as noting the odd, even pattern within the multiples.



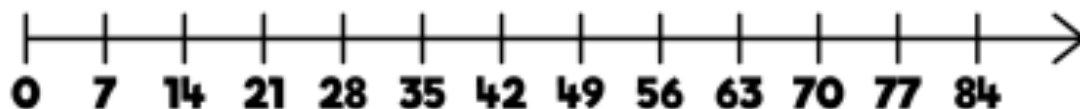
Skill: 7 times table

Year: 4



7	14	21	28	35
42	49	56	63	70

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples both forwards and backwards, supported by a number line or a hundred square. The seven times table can be trickier to learn due to the lack of obvious pattern in the numbers, however they already know several facts due to commutativity. Children can still see the odd, even pattern in the multiples using number shapes to support.

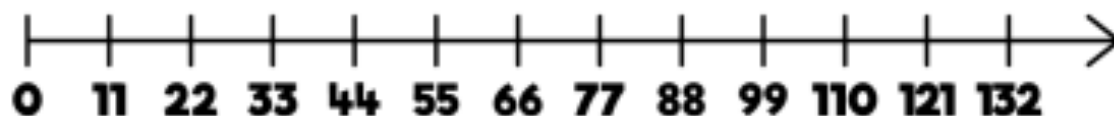
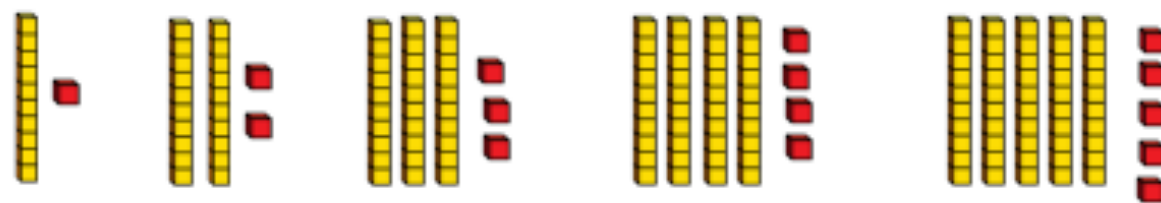


Skill: 11 times table

Year: 4

11	22	33	44	55	66
77	88	99	110	121	132

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the eleven times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support. Also consider the pattern after crossing 100

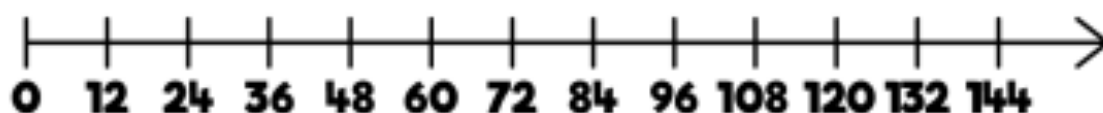
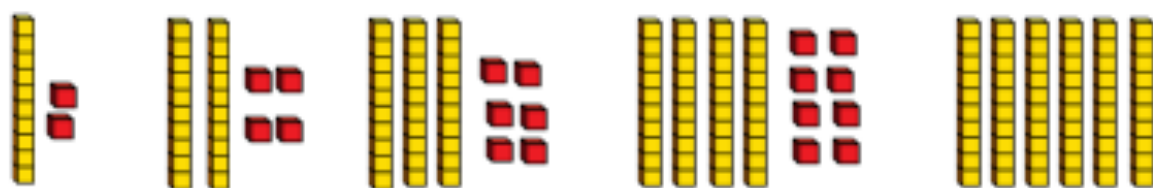


Skill: 12 times table

Year: 4

12	24	36	48	60
72	84	96	108	120
132	144			

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the 12 times table, using manipulatives to support. Make links to the 6 times table, seeing how each multiple is double the sixes. Notice the pattern in the ones within each group of five multiples. The hundred square can support in highlighting this pattern.



Glossary

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor