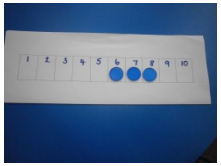
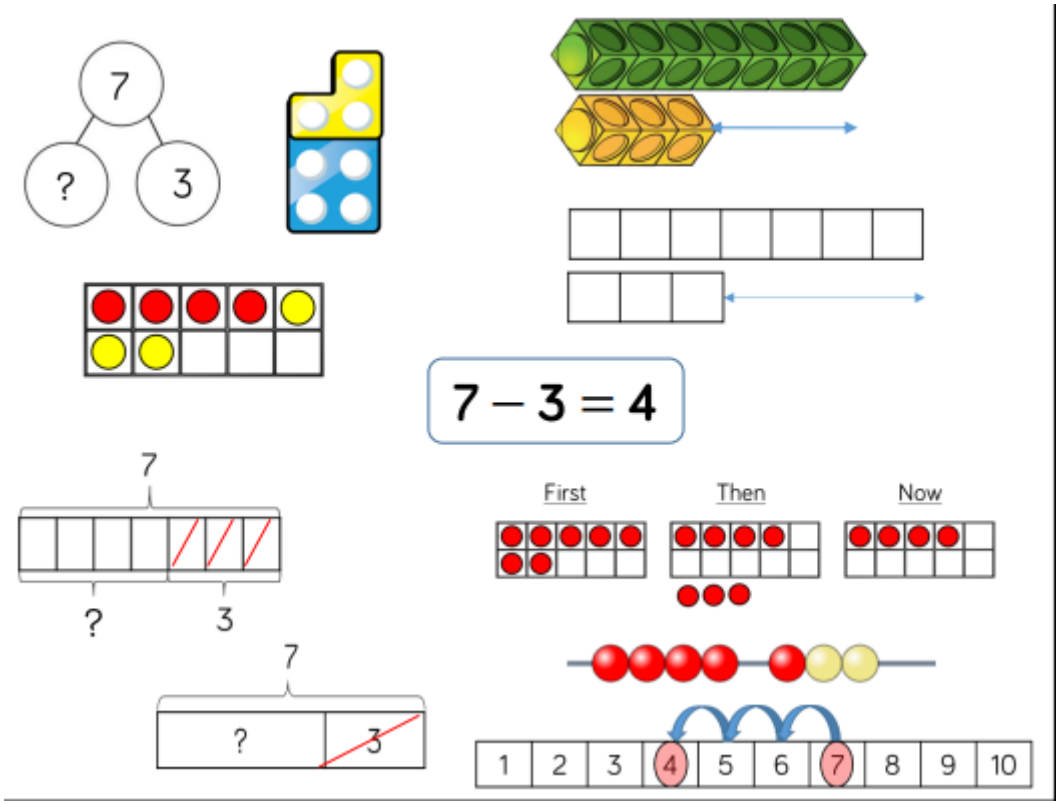
THURNBY MEAD
PRIMARY ACADEMY

Calculation Policy

Subtraction

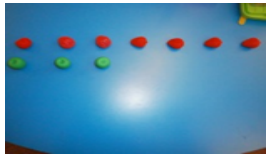
Progression of skills	How to do it
Subtraction	**There are 5 structures for subtraction which need to be introduced to the children** **Reduction: Counting back** **Partitioning: Take away** **Comparison: Numbers are compared (finding the difference)** **Inverse of addition** **The complement of a set (How many counters are yellow and how many are not yellow) This is mainly used in data handling – carroll diagrams etc.** **There is an over reliance on the partitioning structure to model subtraction – it is an important aspect of subtraction but you need to ensure that this is not the only structure taught to the children.**
Oral counting **Count back from x to y** **(Reduction structure)** **Begin to relate subtraction to take away (Partitioning structure)**	Sing songs where you are counting backwards e.g. five little men in a flying saucer. Play games which involve counting back and practical activities to begin to use appropriate vocabulary. Use objects to demonstrate how one is taken away each time. Children to use objects or their fingers to take things away - also show this a number story

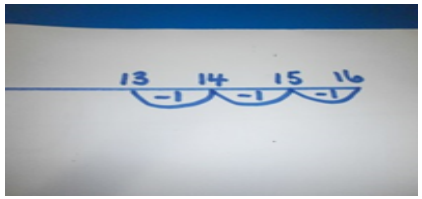
Subtract two 1-digit to 10	 5 take away 1 equals 4  Find one less than a given number – ensure you use the language of one less so the children can relate it to take away 9 – 3 Use equipment for the larger number only and remove the smaller number. Physically remove objects one at a time and find out how many are left. Recognise that there are fewer objects when objects are removed from a set. Use a variety of equipment – Numicon covers are good for this. (Partitioning Structure) Use a number track – child counts back from the first number to the second. (Reduction Structure)  Using double sided counters drop them and the children have to identify how many are yellow and how many are not yellow (complements of a set structure) Use a printed number line  Model to the children using the correct terminology e.g. 5 minus 1 equals 4. Ensure the children understand the language before using the symbols

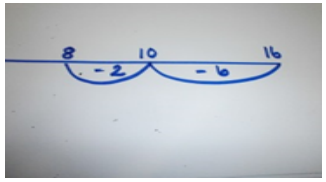


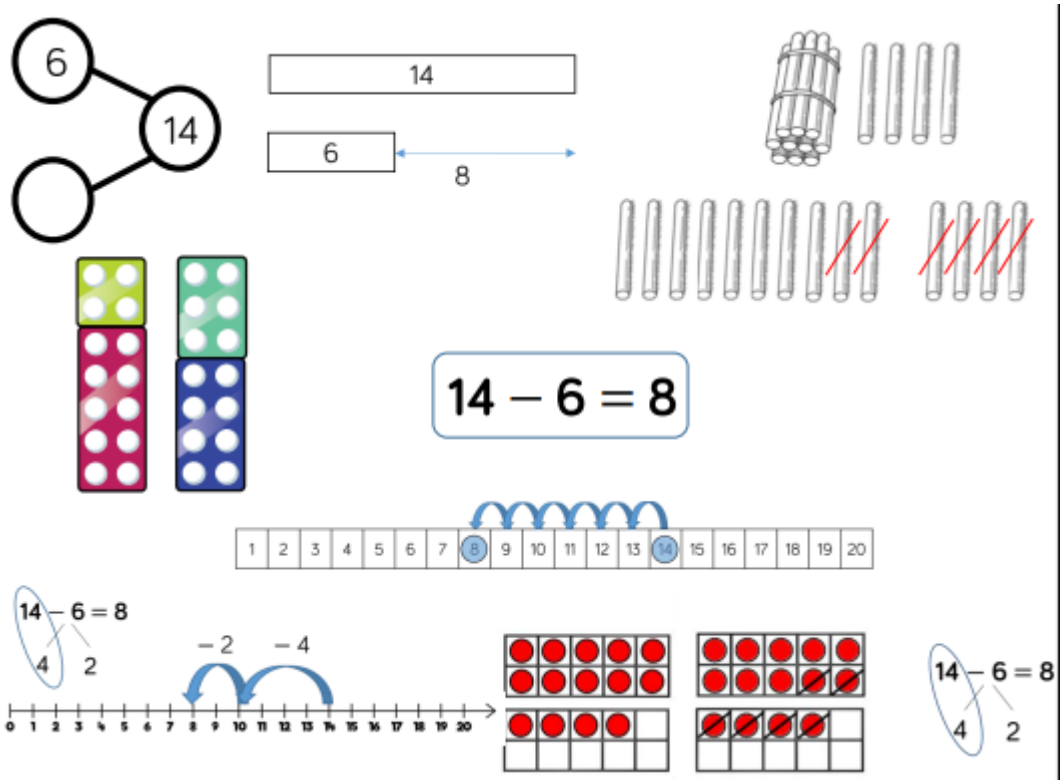
Part whole models, bar models, ten frames and Numicon support partitioning. Tens frames, number tracks, single bar models and bead strings support reduction. Cubes and bar models with 2 bars can support finding the difference.

Subtract by finding the difference (comparison structure)	Use structured apparatus such as Cuisenaire to use the language of finding the difference e.g. the black is shorter than the orange.  The difference between the black and orange is green  Use Numicon to show the difference. The difference between green and red is yellow. The difference between 8 and 5 is 3.   Find 2 numicon tiles that show a difference of 2. How many different ways can you find? 

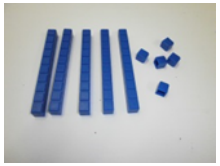
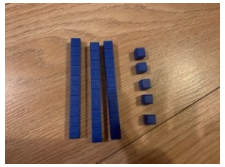
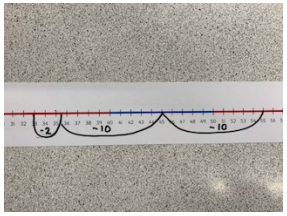
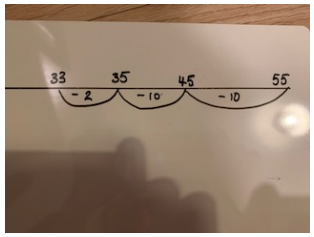
	Use objects to show the difference between 2 numbers 

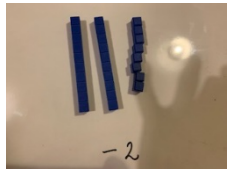
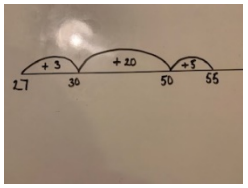
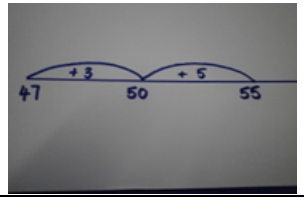
Subtract 1 and 2-digit numbers to 20	Use a printed number line to show the difference between 2 numbers.  <u>Not bridging tens</u> 16 – 3 Use equipment such as base 10 and Numicon. Highlight the link for the ones digit as part of known bonds. E.g. if 6 – 3 = 3 we also know that 16 – 3 = 13 Get equipment for the larger number using knowledge of place value e.g. 1 ten and 6 ones  Use a printed number line Working out on a 100 square – this would only be appropriate if the children are familiar with how a hundred square works Drawing your own number line 

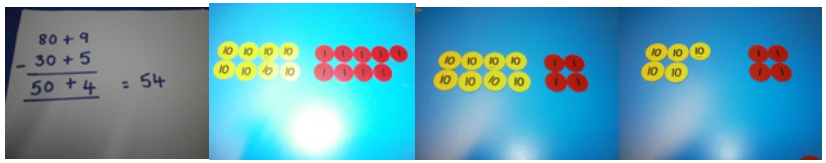
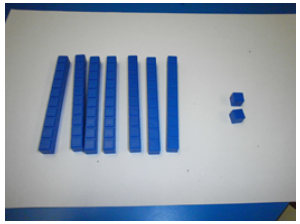
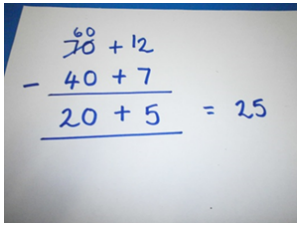
	<u>Bridging tens</u> 16 – 8 Use equipment to get out the largest number and going back to the multiple of 10 and then the rest of the 8.  Efficient jumps on a number line or 100 square Drawing your own number line and doing it in 2 jumps – using known number bonds to support this 



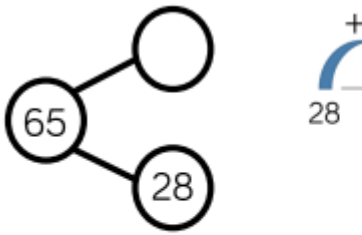
When subtracting one-digit numbers that cross 10, it is important to highlight that ten ones equal one ten.

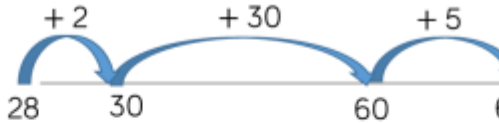
Subtracting two 2-digit numbers	<u>Not bridging 10</u> 55 – 22 Using equipment such as Numicon or base 10, making sure to remove the tens first and then the ones    On a printed number line making sure to count back the tens first and then the ones  Using a hundred square Drawing your own number line, 2 jumps of ten and then the ones  Drawing own number line, a jump of 20 and a jump of 2 ones

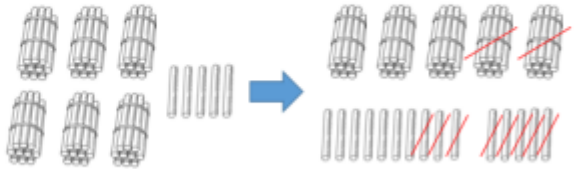
	<u>Bridging 10</u> 55 - 27 Using equipment such as Numicon or base ten     Drawing your own number lines, subtracting the tens digit first and then doing the bonds jumps (55 – 20 - 5 – 2). Drawing your own number line, subtracting the tens digit first - subtracting 20 rather than 2 jumps of 10 and then the ones digit.  55 – 47 Use a number line to show this by counting on from the smallest number to the largest number. 

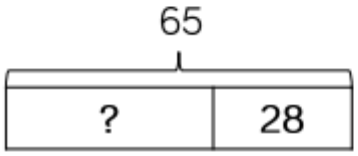
Subtracting 2 digit numbers by finding the difference	Partitioned Column subtraction No exchange involved 89 -35 = 54 Do this alongside equipment so the children can see the link between the equipment and a more formal written method  With exchange 72 – 47 Introduce exchanging through practical subtraction. Make the larger number with the base 10, and then subtract 47 from it. They will need to exchange a 10 for ten units. Then subtract 7 followed by the 4 tens. Show the written method alongside the equipment. When learning to exchange it is important that the children have the opportunity to partition and deconstruct numbers in different ways so that the pupils understand that when you exchange the number is still the same i.e. 72 = 60 + 12 = 50 + 22. Emphasise that the value hasn't changed; we have just partitioned/deconstructed it in a different way.   

	Once pupils are secure with the understanding of exchanging they can use the partitioned column method to subtract any 2 and 3 digit numbers.

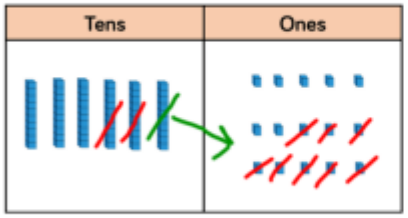


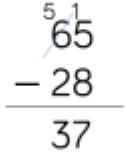


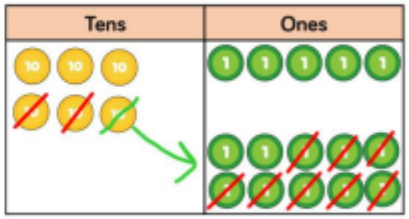


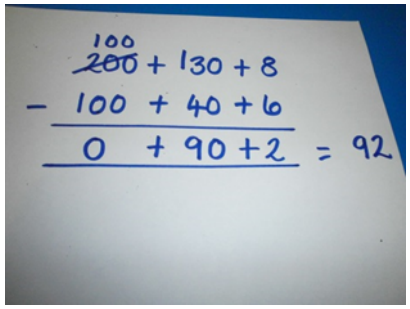


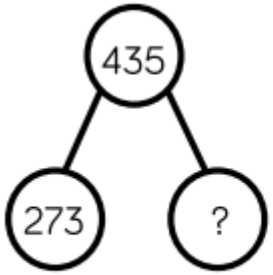
$65 - 28 = 37$

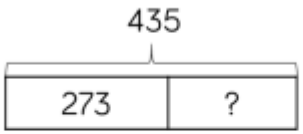


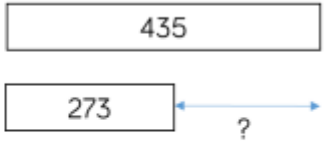




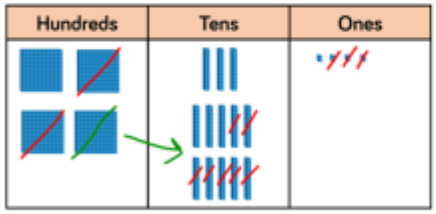
Subtracting 2 and 3 digit numbers **Subtracting by counting on**	At this stage encourage the children to use the formal column method when calculated alongside straws, base-ten or place value counters. As numbers become larger, straws become less efficient. Children can also use a blank number line to count on to find the difference. Encourage them to jump to multiples of 10 to become more efficient. 238 – 146  Continue counting on as a mental strategy for numbers that are close together and for the numbers that are nearly multiples of 10, 100 and 1000 e.g. 501 – 495 Compact column subtraction Children need to have a very secure understanding of place value and knowledge of number facts to move onto this method



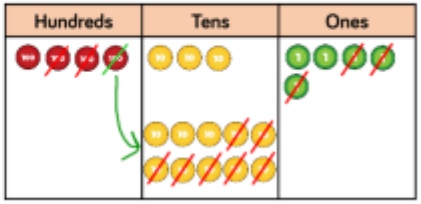


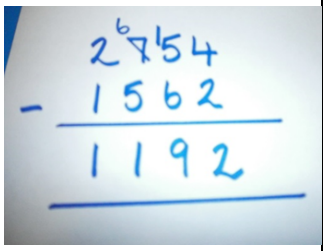


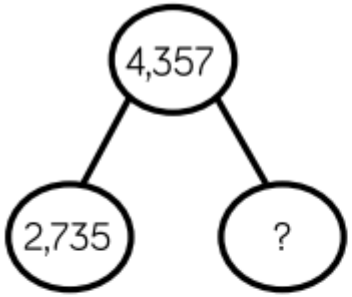
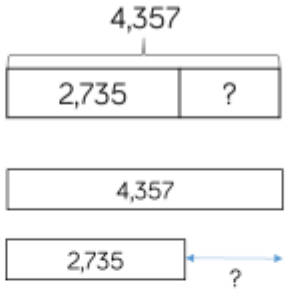
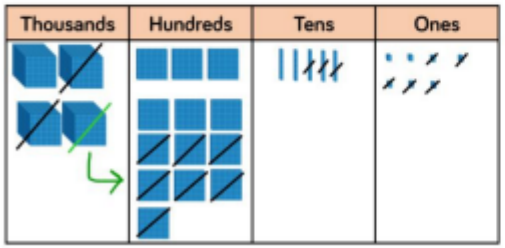
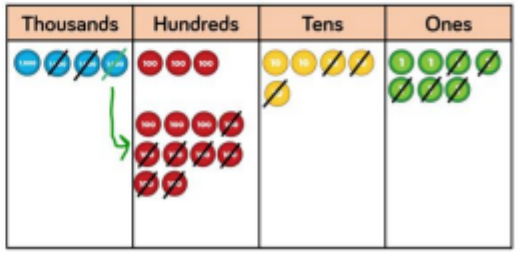
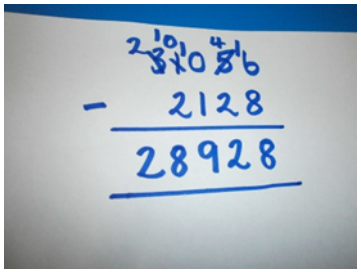
$$435 - 273 = 262$$



$$\begin{array}{r} {}^{3}\cancel{4}{}^{1}35 \\ -\ 273 \\ \hline 262 \end{array}$$

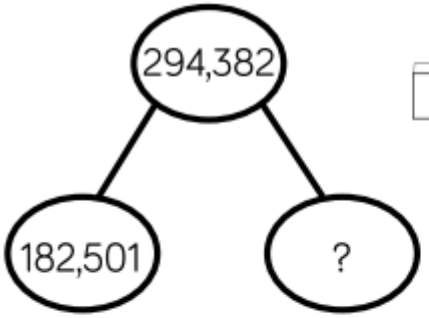


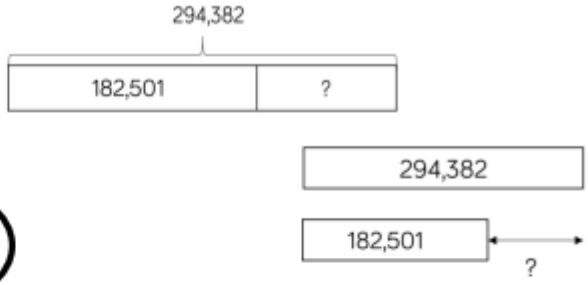
Subtract with up to 4 digits	Base ten and place value counters are the most effective manipulatives when subtracting numbers with up to 3 digits. Ensure children write out their calculation alongside any concrete resources so they can see the link with the written column method. Plain counters on a place value grid can also be used.to support learning. 2754 – 1562  To introduce the compact method, ask the children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask the pupils to consider how it relates to the method they know, what is similar and what is different? This will develop their understanding of it.

Subtract with at least 4-digit numbers	  3 1 4357 − 2735 1622 **4,357 − 2,735 = 1,622**   31056 – 2128 

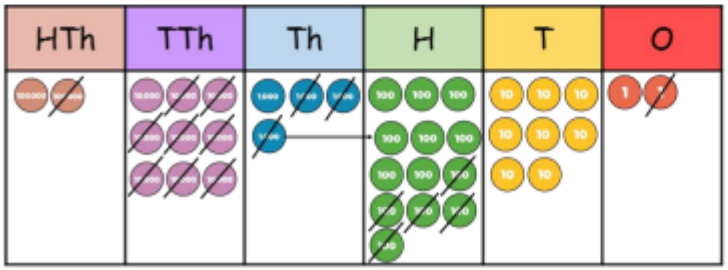
Base ten and place value counters are the most effective manipulatives when adding numbers with up to 3 digits. Ensure children write out their calculation alongside any concrete resources so they can see the link with the written column method. Plain counters on a place value grid can also be used.to support learning.

Subtract with up to 3 decimal places





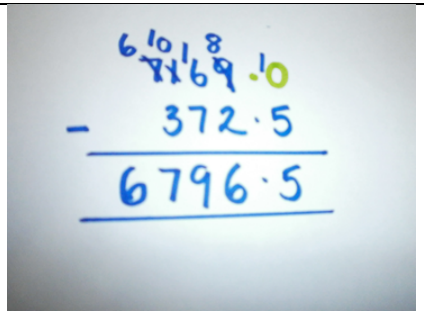
294,382 − 182,501 = 111,881



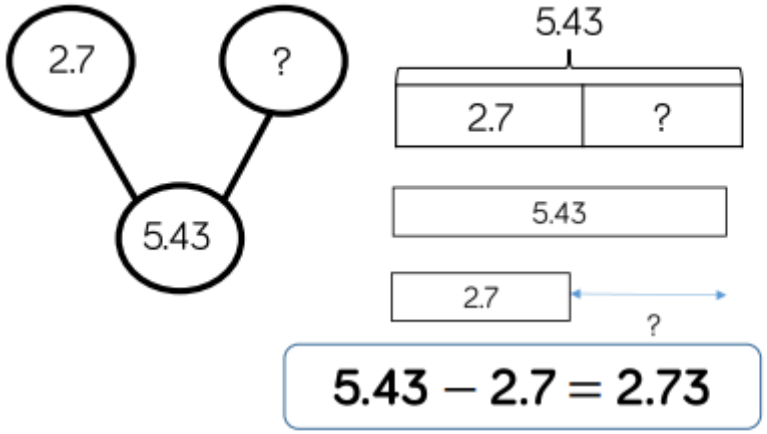
	2	9	$^{3}\not{4}$	$^{1}3$	8	2
−	1	8	2	5	0	1
	1	1	1	8	8	1

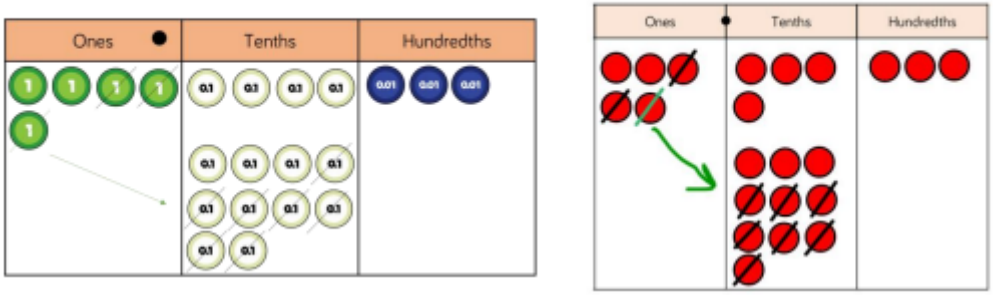
Place value counters or plain counters on a place value grid are the most effective concrete resources when subtracting numbers with more than 4 digits. At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point
Add a zero in any empty decimal places to aid understanding of what to subtract in that column.



Create lots of opportunities for subtracting and finding differences with money and measures





Place value counters and plain counters on a place value grid are the most effective manipulatives when subtracting decimals with 1,2 and then 3 decimal places. Ensure children have experience of adding decimals with a variety of decimal places. This includes putting it into context when adding money and other measures.

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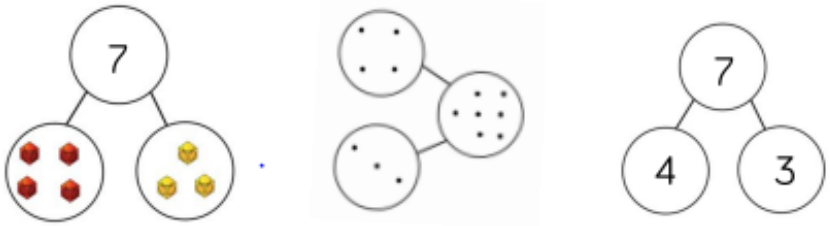
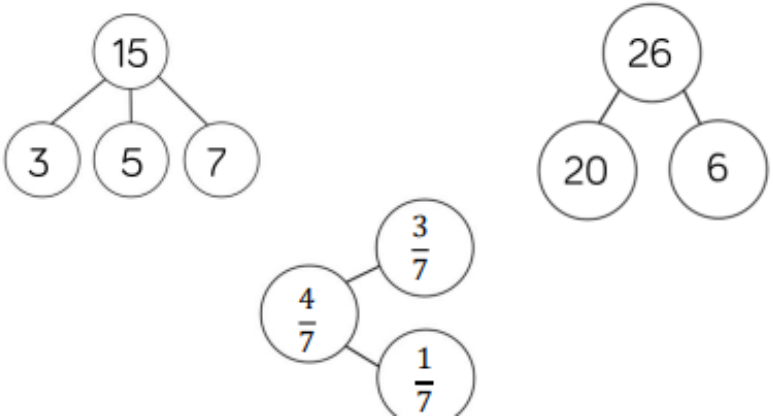
Benefits of Equipment Models and Images

Part-Whole Model



$7 = 4 + 3$
$7 = 3 + 4$

$7 - 3 = 4$
$7 - 4 = 3$



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

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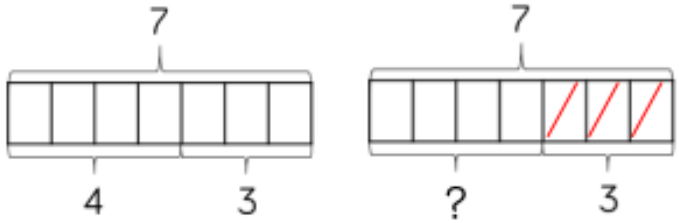
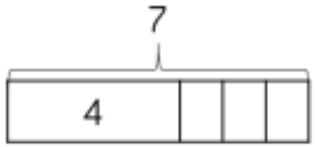
Bar Model (single)

Concrete

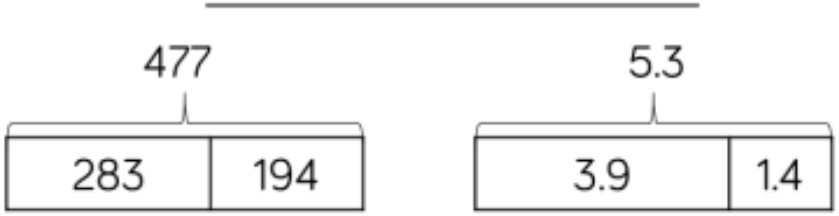
Discrete



Combination



Continuous



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

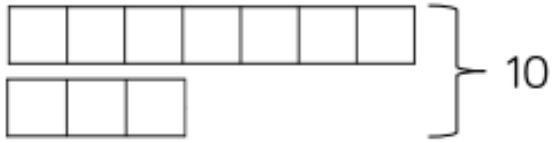
Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

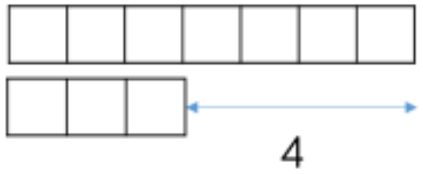
THURNBY MEAD
PRIMARY ACADEMY

Bar Model (multiple)

Discrete

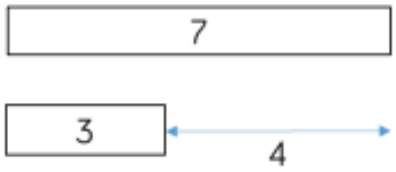


$7 + 3 = 10$

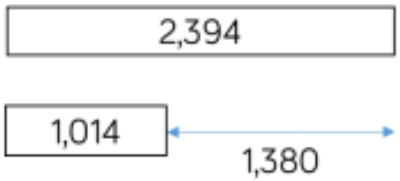


$7 - 3 = 4$

Continuous



$7 - 3 = 4$



$2{,}394 - 1{,}014 = 1{,}380$

Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

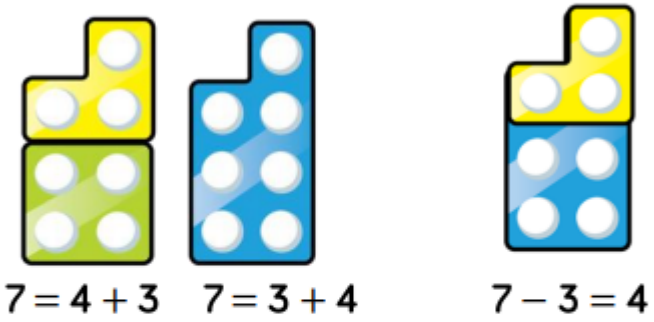
Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

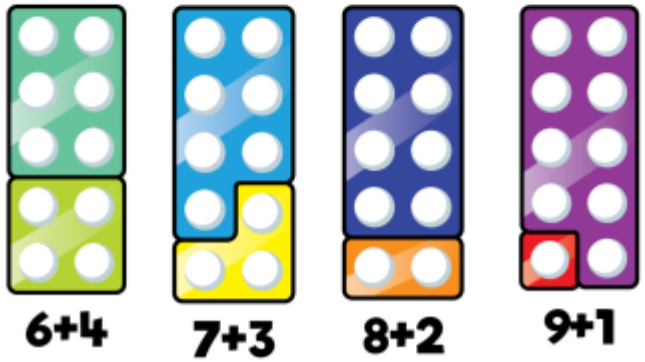
Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

THURNBY MEAD
PRIMARY ACADEMY

Number Shapes





Benefits

Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

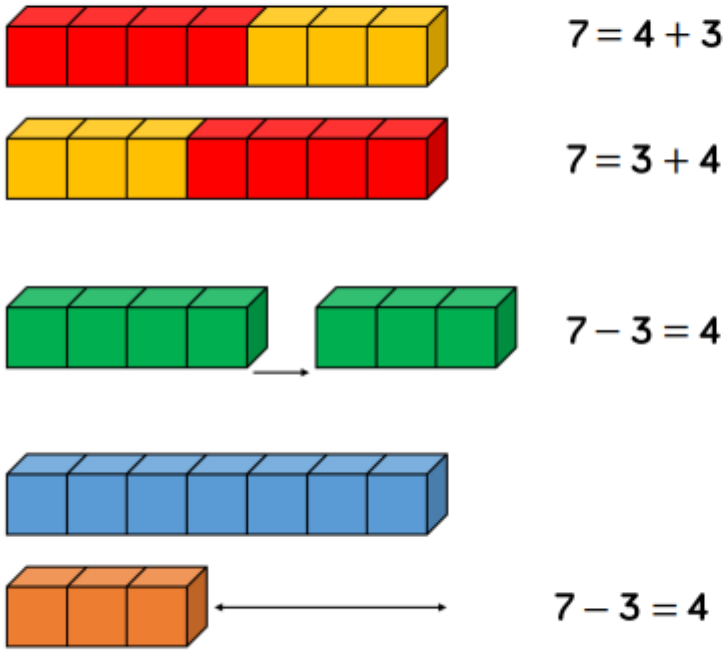
When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

THURNBY MEAD
PRIMARY ACADEMY

Cubes



$7 = 4 + 3$

$7 = 3 + 4$

$7 - 3 = 4$

$7 - 3 = 4$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

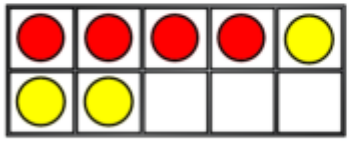
When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

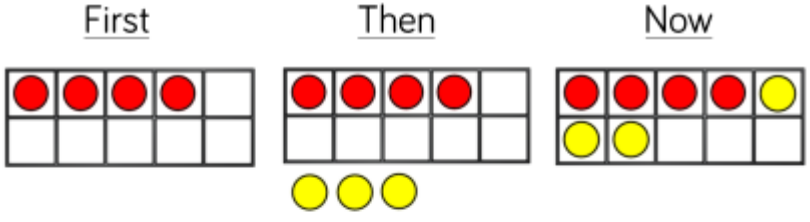
THURNBY MEAD
PRIMARY ACADEMY

Ten Frames (within 10)

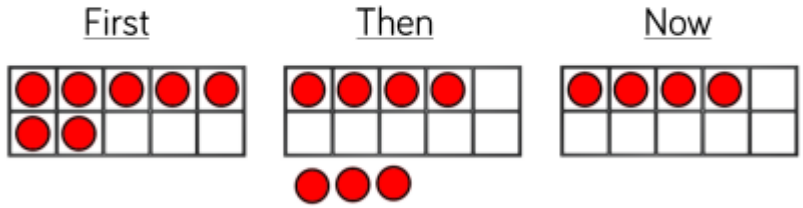


$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

4 is a part.
3 is a part.
7 is the whole.



$4 + 3 = 7$



$7 - 3 = 4$

Benefits

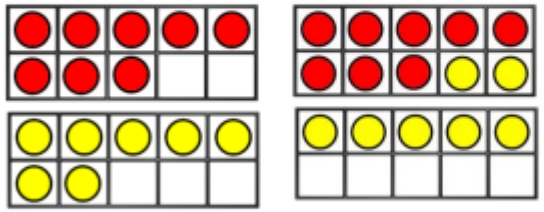
When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

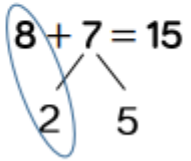
Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

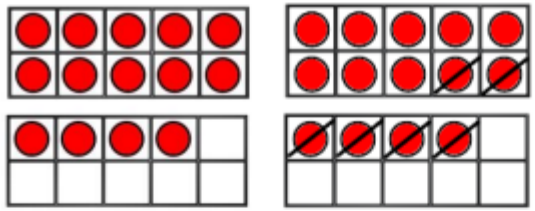
Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

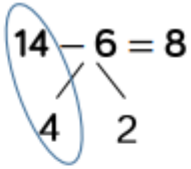
THURNBY MEAD
PRIMARY ACADEMY

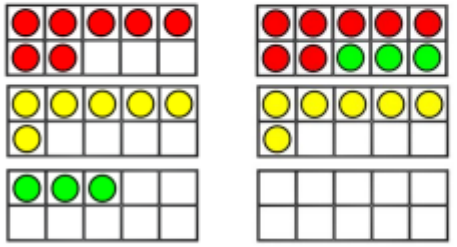
Ten Frames (within 20)

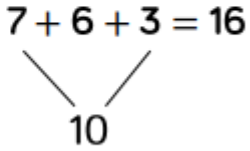












Benefits

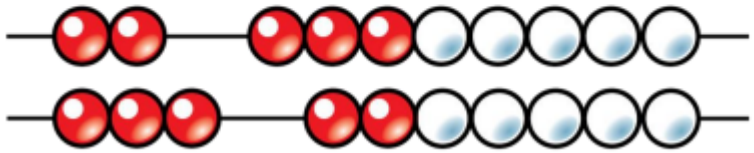
When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

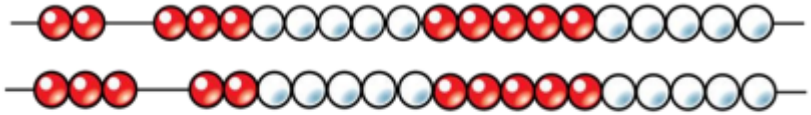
When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

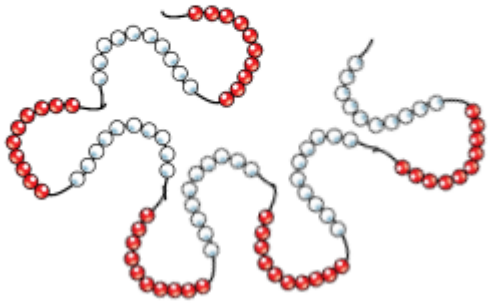
When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

THURNBY MEAD
PRIMARY ACADEMY

Bead Strings







Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10.
They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. 2 + 8 = 10, move one bead, 3 + 7 = 10.

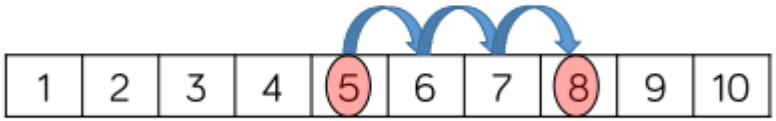
Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

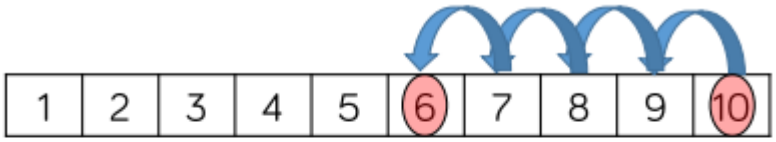
THURNBY MEAD
PRIMARY ACADEMY

Number Tracks

$5 + 3 = 8$



$10 - 4 = 6$



$8 + 7 = 15$

Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

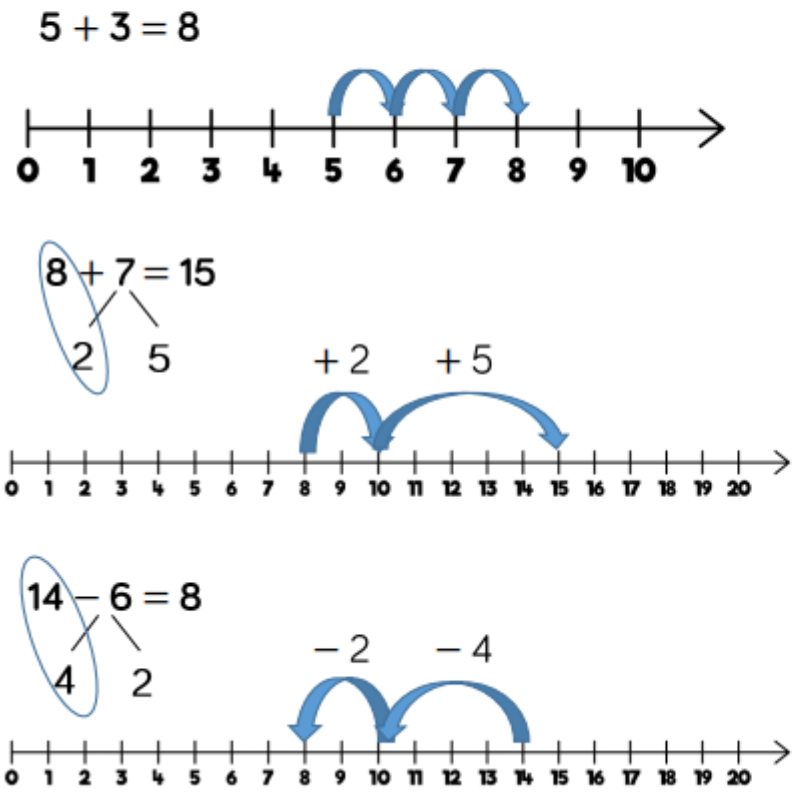
When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

THURNBY MEAD
PRIMARY ACADEMY

Number Lines (labelled)



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

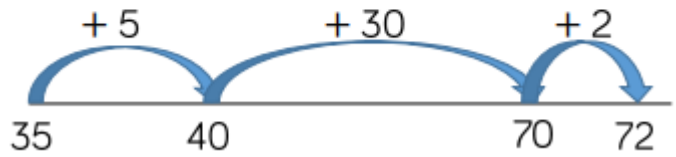
Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

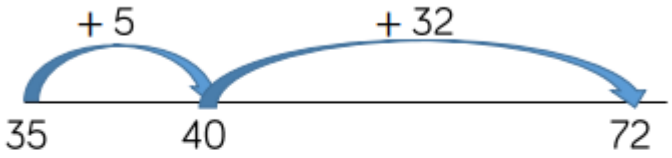
THURNBY MEAD
PRIMARY ACADEMY

Number Lines (blank)

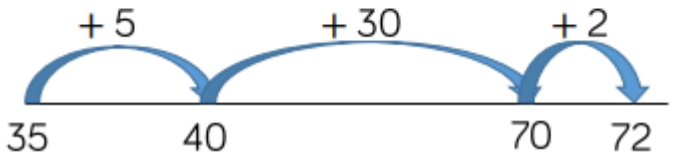
35 + 37 = 72



35 + 37 = 72



72 − 35 = 37



Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

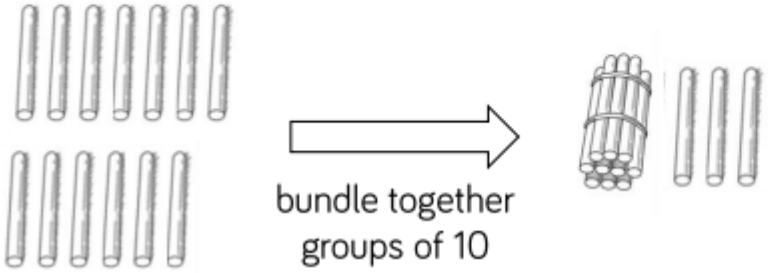
Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

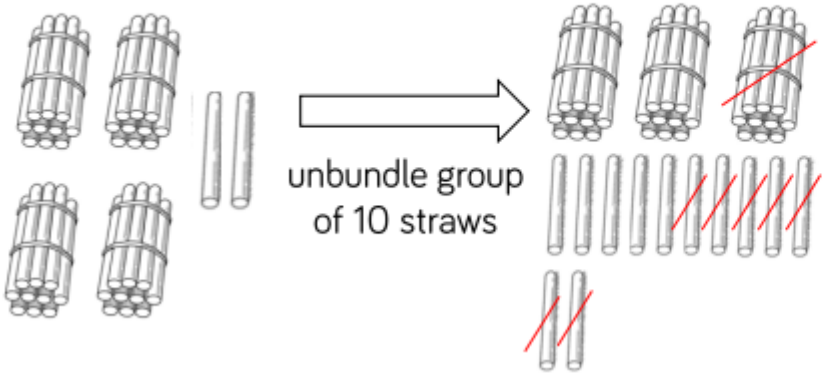
THURNBY MEAD
PRIMARY ACADEMY

Straws

$7 + 6 = 13$



$42 - 17 = 25$



Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

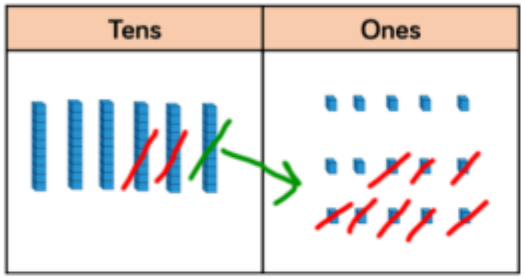
When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

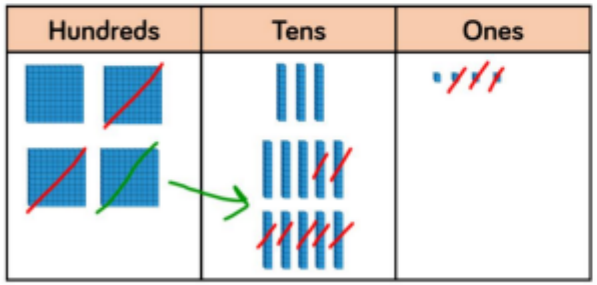
Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

THURNBY MEAD
PRIMARY ACADEMY

Base 10/Dienes (subtraction)



$$\begin{array}{r} {}^{5}\not{6}\,{}^{1}\not{5} \\ -\ 2\ 8 \\ \hline 3\ 7 \end{array}$$



$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}\not{3}\,5 \\ -\ 2\ 7\ 3 \\ \hline 2\ 6\ 2 \end{array}$$

Benefits

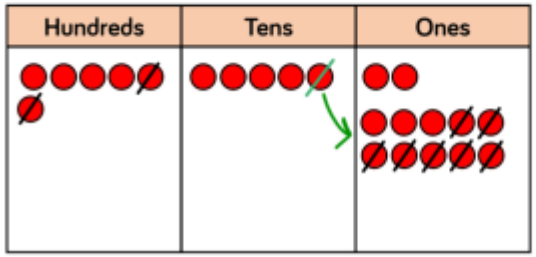
Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

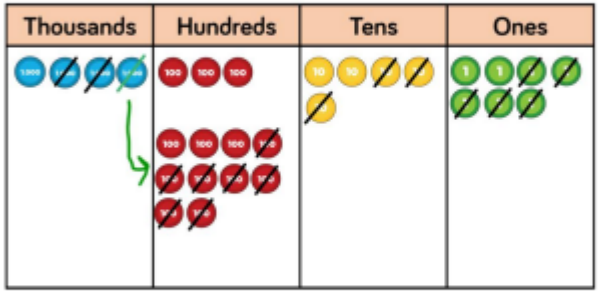
This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

THURNBY MEAD
PRIMARY ACADEMY

Place Value Counters (Subtraction)



$$\begin{array}{r} {}^{4}\not{6}\,{}^{1}\not{5}2 \\ -\ 207 \\ \hline 445 \end{array}$$



$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

THURNBY MEAD
PRIMARY ACADEMY

Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative – numbers can be added in any order.

Complement – in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange – Change a number or expression for another of an equal value.

Minuend – A quantity or number from which another is subtracted.

Partitioning – Splitting a number into its component parts.

Reduction – Subtraction as take away.

Subitise – Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total – The aggregate or the sum found by addition.